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AN INTRODUCTION ON FUZZY SEMI–SUBMAXIMAL SPACES AND APPLICATIONS TO MEDICAL DIAGNOSIS

[1]D. NIRAJA, [2]Dr. D.VIJAYAN, [3]N. MYTHILI

[1]Ph.D Research Scholar,[2]Associate Professor,[3]Ph.D. Scholar

Department of Mathematics,

Muthurangam Government Arts College (Autonomous), Vellore–632 002.

Affiliated to Thiruvalluvar University,Serkadu, Vellore-632 115,Tamil Nadu, India.

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Abstract

Present paper studies the concept ofsemi–submaximal spacein fuzzy topological spaces. Variousproperties in fuzzy semi–submaximal spaces were investigated. To be proved that asemi-submaximal space becomes semi-nowhere dense setof topology in fuzzy. To show thatthe semi-submaximal space gives the result ofsemi-open and semi-closed sets in fuzzy topology.The condition explores fuzzy semi–submaximal spaceto fuzzy semi–dense and fuzzy G_δ sets. Additionally, condition is analyzed under which the fuzzy submaximal space attains fuzzy semi-submaximal space. Further,thestudy of fuzzy semi-submaximal interrelate with fuzzy semi-baire space. Fuzzy semi-submaximal spaces and related fuzzy topological spaces are in submaximal fitness test to model the uncertainty, and ambiguity in physiological data (like heart rate ((HR), walk test). It is used to enhance the accuracy of fitness.

1. *INTRODUCTION*

Zadeh, L.A. in 1965 [10], was introduced the studyoffuzzy sets. By giving partial membership to each element under consideration in real life situations.The interpretation of fuzzy sets is influenced not only by the concept itself but also by the context in which it is applied by the mathematicians to fuzziness structures. Chang, C.L. in 1968 [6], was developed the generalization of topological spaces in fuzzy by using the fuzzy sets. Levine,N. in 1963 [7], introduced the semi-open sets and Azad, K.K.[3], further studied and expanded on these ideas in fuzzy.

Bourbaki [5], has been explored the concept of submaximal spaces in general topology. In general topologies the class of submaximal spaces has gained significant interest. It was first studied systematic of submaximal spaces by ArhangelSkiand,A.V. and Collins,P.J, [2]. Balasubramanian, G [4] gave the idea of extending submaximal spaces in fuzzy topology.Fuzzy semi–submaximal spaces of variouspropertieswerediscussed.Additionally, it was studiedthat fuzzy semi-submaximal spaces interrelate with other fuzzy semi-topological spaces.

2. *PRELIMINARIES*

Definition: 2.1-Fuzzy Topology[6]

The family of fuzzy topology ρin fuzzy sets Ωsatisfies the following three conditions

1. $0_\Omega, 1_\Omega \,\varepsilon\, \rho$
2. If x, y ε ρ, then $x \wedge y \,\varepsilon\, \rho$
3. $x_i \,\varepsilon\, \rho$, for each $i \,\varepsilon\, I$, then $\vee_{i \varepsilon I} x_i \,\varepsilon\, \rho$

Here the fuzzy set Ω is said to be fuzzy topology then the ordered pair (Ω, ρ) is a fuzzy topological space (in short FTS). In a fuzzy topology allmembersof ρis called a fuzzy ρ–open sets and its complement of fuzzy ρ-open sets is fuzzyρ-closed sets.

Definition: 2.2-Fuzzy Semi-Topology[8]

Fuzzy semi-topology is family of fuzzy set (Ω, ρ_s)whichholds the below three axioms:-

1. $0_\Omega, 1_\Omega \,\varepsilon\, \rho_s$,
2. Let $\chi, \nu \,\varepsilon\, \rho_s$, then $\chi \wedge \nu \,\varepsilon\, \rho_s$
3. If $\chi_i \,\varepsilon\, \rho_s$, for each $i \,\varepsilon\, I$, then $\vee_{i \varepsilon I} \chi_i \,\varepsilon\, \rho_s$

A fuzzy set Ω is said to have fuzzy semi-topology and the pair of (Ω, ρ_s) is called a fuzzy semi-topological space.In fuzzy semi-topology ρ_s each member is known as fuzzy ρ_s-open set.The fuzzy ρ_s-closed set is their complements of fuzzy ρ_s-open set.

Definition: 2.3-[6]

If fuzzy setsσ and η are in Ω,then for all a∈ Ω,

1. $\sigma = \eta \Leftrightarrow \sigma(a) = \eta(a)$,

2. $\sigma \le \eta \Leftrightarrow \sigma(a) \le \eta(a)$,

3. $\psi = \sigma \vee \eta \Leftrightarrow \psi(a) = \max\{\sigma(a), \eta(a)\}$,

4. $\delta = \sigma \wedge \eta \Leftrightarrow \delta(a) = \min\{\sigma(a), \eta(a)\}$,

5. $\upsilon = \sigma^c \Leftrightarrow \upsilon(a) = 1 - \sigma(a)$.

In Ω the fuzzy sets of union and intersection is denoted by $\partial = \vee_i \sigma_i$ and $\Phi = \wedge_i \sigma_i$ and it is defined as $\partial(a) = \sup_i\{\sigma_i(a), a \in \Omega\}$ and $\Phi(a) = \inf_i\{\sigma_i(a), a \in \Omega\}$.

A fuzzy set 0_Ω and 1_Ω is defined as $0_\Omega(a) = 0$ and $1_\Omega(a) = 1$, for all $a \in \Omega$.

Definition:2.4[1]

A fuzzy semi-open set is if $\sigma \le cl_s(int_s(\sigma))$ in a fuzzy semi-topological space.

Definition:2.5

A fuzzy set σ in a fuzzy semi-topological space (Ω, ρ_s) if $int_s(cl_s(\sigma)) \le \sigma$, then it is a fuzzy semi-closed set. [1]

Definition:2.6 [4]

(i) $cl_s(\sigma) = \wedge\{\eta/\sigma \le \eta$, η is fuzzy semi-closed set of $\Omega\}$

(ii). $int_s(\sigma) = \vee\{\eta/\eta \le \sigma$, here η is a fuzzy semi-open set of $\Omega\}$

Lemma:1[4]

Fuzzy semi-topological space Ω of a fuzzy set σ

1. $int_s(1-\sigma) = 1 - cl_s(\sigma)$

2. $cl_s(1-\sigma) = 1 - int_s(\sigma)$

Definition:2.7

A fuzzy semi-topological space(Ω, ρ_s) of a fuzzy set σ is

i) if there exists no fuzzy semi-closed set η such that $\sigma < \eta < 1$. i.e., $cl_s(\sigma)=1$ is fuzzy semi-dense set [1] -

ii) fuzzy semi-nowhere dense set isif there does not exists a non-zero fuzzy semi-open set η in (Ω, ρ_s) such that $\mu < cl_s(\sigma)$. i.e., $int_s(cl_s(\sigma)) = 0$. [2]

Definition:2.8

If $\sigma \in \rho$ in a fuzzy topological space (Ω, ρ) is fuzzy submaximal space if for every fuzzy set σsuch that $cl(\sigma) = 1$. Otherwise, if all fuzzy dense set is fuzzy open set, then it is a fuzzy submaximal space.[9]

3. *Fuzzy Semi-Submaximal Spaces*

Definition:3.1-FSSS

A fuzzy semi-submaximal space is if every fuzzy semi-dense set is fuzzy semi-open set such that $\sigma \le cl_s(int_s(\sigma))$ in fuzzy semi-topological space(Ω, ρ_s)

Example-3.2

SupposeH, M and N are fuzzy sets on $\Omega = \{c, d, e\}$ and it is defined below:

$H: \Omega \to [0,1]$ is termed as $H(c) = 0.5$; $H(d) = 0.4$; $H(e) = 0.5$

$M: \Omega \to [0,1]$ is defined as $M(c) = 0.6$; $M(d) = 0.4$; $M(e) = 0.3$

N: Ω → [0, 1] is takenas N(c) = 0.5; N(d) = 0.5; N(e) = 0.5

Then a fuzzy semi-topology $\rho_s$is = {0, H, M, N, $H \vee M, H \vee N, M \vee N, H \wedge M$, 1}inΩ.

Bycalculatingthe fuzzy semi–open sets are {0, H, M, N, $H \vee M, H \vee N, M \vee N, H \wedge M$, 1} and fuzzy semi–closed sets are {0, 1-H, 1-M, 1-N, $1-(H \vee M)$, $1-(H \vee N)$, $1-(M \vee N)$, $1-(H \wedge M)$}. Therefore, all fuzzy semi–dense setsH, M, N are fuzzy semi–open sets. Hence (Ω, ρ_s) is a fuzzy semi–submaximal space.

Example-3.3

Let Ω = {p, q, r}. Let Γ, Π and Ψtakes the values on Ω as follows:

Γ: Ω → [0,1] is given byΓ(p) = 0.7; Γ(q) = 0.8; Γ(r) = 1

Π: Ω → [0,1] is termed asΠ(p) = 1; Π(q) = 0.9; Π(r) = 0.5

Ψ: Ω → [0,1] is defined byΨ(p) = 0.8; Ψ(q) = 1; Ψ(r) = 0

Here ρ_s={0, Γ, Π, Ψ, $\Gamma \vee \Pi, \Gamma \vee \Psi, \Pi \vee \Psi, \Gamma \wedge \Pi, \Gamma \wedge \Psi, \Pi \wedge \Psi$, 1} are fuzzy semi-topologyΩ.

Bysolving, the fuzzy semi–open sets are {0, Γ, Π, Ψ, $\Gamma \vee \Pi, \Gamma \vee \Psi, \Pi \vee \Psi, \Gamma \wedge \Pi, \Gamma \wedge \Psi, \Pi \wedge \Psi$, 1} then fuzzy semi–closed sets are {0, 1-Γ, 1-Π, 1-Ψ, $1-(\Gamma \vee \Pi)$, $1-(\Gamma \vee \Psi)$, $1-(\Pi \vee \Psi)$, $1-(\Gamma \wedge \Pi)$, $1-(\Gamma \wedge \Psi)$, $1-(\Pi \wedge \Psi)$, 1}. Hence, Γ, Π and Ψ are fuzzy semi-dense sets but does not satisfy fuzzy semi–open set. Therefore,(Ω, ρ_s) is not a fuzzy semi–submaximal space.

4. *Characterizations of Fuzzy Semi–Submaximal Spaces*

Proposition-4.1

If int_s(σ) = 0 where σ is a fuzzy set definedin(Ω, ρ_s), then σis afuzzy semi-nowhere dense set in a fuzzy semi-submaximal space (Ω, ρ_s).

Proof:

Assume that int_s(σ) = 0, where σfuzzy set defined in (Ω, ρ_s). Then 1−int_s (σ) = 1−0 = 1. Bya givenlemma 1, cl_s(1−σ) = 1. Given (Ω, ρ_s) is fuzzy semi-submaximal spaceit follows that1-σ is fuzzy semi-open settheno is fuzzy semi-closed set in (Ω, ρ_s). Now by usingcl_s(σ) = σ implies $int_s cl_s$(σ)= int_s(σ)=0.Hence, σexplores fuzzy semi-nowhere dense set in (Ω, ρ_s).

Proposition-4.2

In fuzzy semi-submaximal space, iffuzzy set σ is a fuzzy semi-nowhere dense set, then the complement of σ is fuzzy semi-open set in (Ω, ρ_s).

Proof:

Suppose σ be a fuzzy semi-nowhere dense set in (Ω, ρ_s) then by definition int_s(cls(σ))=0. But int_s(σ) ≤$int_s cl_s$(σ)implies int_s(σ) = 0. So 1−int_s(σ) = 1 by the lemma 1, cl_s(1−σ) = 1. Since (Ω, ρ_s) is fuzzy semi-submaximal space then the complement of σ is fuzzy semi-open set in (Ω, ρ_s).

Proposition-4.3

Let (Ω, ρ_s) is fuzzy semi-submaximal spacetheint$_s$(σ)=0 if and only if int$_s$cl$_s$(σ) =0, then σis fuzzy set defined on (Ω, ρ_s).

Proof:

Suppose int$_s$(σ)=0, where σ is a fuzzy set defined by the fuzzy semi-topological space (Ω, ρ_s) byproposition 4.1int$_s$cl$_s$ (σ) = 0. Since cl$_s$(σ) = σ, then int$_s$(σ) = 0. Therefore, int$_s$cl$_s$(σ) = 0.

Conversely, suppose int$_s$cl$_s$(σ) = 0 in (Ω, ρ_s). Now $\sigma \leq \mathrm{int}_s\mathrm{cl}_s(\sigma)$ implies $\mathrm{int}_s(\sigma) \leq \mathrm{int}_s\mathrm{cl}_s(\sigma)$ then $\mathrm{int}_s(\sigma) \leq 0$ which is int$_s$(σ) = 0 in (Ω, ρ_s).

Proposition-4.4

Let1-σbea fuzzy semi–nowhere dense set in fuzzy semi-topological space (Ω, ρ_s).Then the interior satisfies int$_s$(1-σ)=0.

Proof:

In a fuzzy semi-topological space (Ω, ρ_s), consider 1-σbea fuzzy semi–nowhere dense set since int$_s$(cls(1-σ))=0. Since $1 - \sigma \leq \mathrm{cl}_s(1 - \sigma)$, it follows that $\mathrm{int}_s(1 - \sigma) \leq \mathrm{int}_s(\mathrm{cl}_s(1 - \sigma)) = 0$. Therefore, int$_s$(1–σ)=0.

Proposition-4.5

Consider (Ω, ρ_s) is a fuzzy semi-topological space.If 1–σ is a fuzzy semi–nowhere dense set (Ω, ρ_s), then 1-(1-σ) is a fuzzy semi–dense set in (Ω, ρ_s).

Proof:

Assume 1–σ is fuzzy semi–nowhere dense set in (Ω, ρ_s). By definition int$_s$(cls(1-σ))=0. But

$$1 - \sigma \leq \mathrm{cl}_s(1 - \sigma)$$
$$\Rightarrow \mathrm{int}_s(1-\sigma) \leq \mathrm{int}_s\mathrm{cl}_s(1-\sigma)$$
$$\Rightarrow \mathrm{int}_s(1-\sigma) = 0$$

Then cl$_s$(σ) =1-int$_s$(1- σ)= 1–0=1 (by a lemma 1). Therefore,σ is fuzzy semi–dense set in (Ω, ρ_s).

Proposition-4.6

The fuzzy set 1–σ is fuzzy semi–nowhere dense set and 1-η is any fuzzy set in fuzzy semi-topological space (Ω, ρ_s), if and only if $(1-\sigma) \wedge (1-\eta)$ is a fuzzy semi–nowhere dense set in (Ω, ρ_s).

Proof:

If 1-η is a fuzzy semi–nowhere dense set in (Ω, ρ_s), then by given definition int$_s$(cls(1-η)) = 0. Now $\mathrm{int}_s\mathrm{cl}_s((1-\sigma) \wedge (1-\eta)) \leq \mathrm{int}_s\mathrm{cl}_s(1-\sigma) \wedge \mathrm{int}_s\mathrm{cl}_s(1-\eta) \leq \mathrm{int}_s\mathrm{cl}_s(1-\sigma) \wedge 0 = 0$ such that $\mathrm{int}_s\mathrm{cl}_s((1-\sigma) \wedge (1-\eta)) = 0$. Hence, $(1-\sigma) \wedge (1-\eta)$ is int$_s$(cls(1-η)) = 0 in (Ω, ρ_s).

Conversely, assume $(1-\sigma) \wedge (1-\eta)$ is a fuzzy semi–nowhere dense set in (Ω, ρ_s), it follows that

$$\mathrm{int}_s\mathrm{cl}_s[(1-\sigma) \wedge (1-\eta)] = 0$$
$$\Rightarrow \mathrm{int}_s\mathrm{cl}_s(1-\sigma) \wedge \mathrm{int}_s\mathrm{cl}_s(1-\eta) = 0$$

Since 1-σ is a fuzzy semi–dense set in (Ω, ρ_s). We get

$$\mathrm{int_s}(1) \wedge \mathrm{int_s cl_s}(1-\eta) = 0$$
$$\Rightarrow 1 \wedge \mathrm{int_s cl_s}(1-\eta) = 0$$

Therefore, $\mathrm{int_s cl_s}(1-\eta) = 0$.

Proposition-4.7

In a fuzzy semi-submaximal space (Ω, ρ_s), If $\mathrm{int_s}\left[\overset{\infty}{\underset{i=1}{\vee}}(\sigma_i)\right] = 0$, where σ_i be a fuzzy semi–nowhere dense setthen (Ω, ρ_s) is a fuzzy semi–volterra.

Proof:

Assume σ_i ε $\rho_s$is a fuzzy semi–nowhere dense setsin a fuzzy semi-submaximal space (Ω, ρ_s). By the known result semi-closure and semi-interior of fuzzy satisfies $cl_s(\sigma_i) = 1$ and $int_s(\sigma_i) = \sigma_i$. Since $\sigma_i$is fuzzy semi-dense and fuzzy semi-open set. Thus$int_s(cls(\sigma_i))=0$ is fuzzy semi-nowhere dense set. Suppose we have $cl_s int_s(\sigma_i)=1$ which implies that 1- $cl_s int_s(\sigma_i) = 0$ gives 1- $cl_s(\sigma_i) = 0$ implies that $cl_s(\sigma_i) = 1$, implies σ_i is fuzzy–semi dense sets in (Ω, ρ_s). Now

$$\mathrm{int_s}\left(\overset{\infty}{\underset{i=1}{\vee}}(\sigma_i)\right) = 0$$
$$\Rightarrow 1 - \mathrm{int_s}\left(\overset{\infty}{\underset{i=1}{\vee}}(\sigma_i)\right) = 1$$

then

$$\mathrm{cl_s}\left(\overset{\infty}{\underset{i=1}{\wedge}}(\sigma_i)\right) = 1$$

Hence, by definition (Ω, ρ_s) is fuzzy semi–volterra space.

Proposition-4.8

If (Ω, ρ_s) is a fuzzy semi–submaximal space, then for any fuzzy set σ defined on Ω, $\mathrm{cl_s}(\sigma) \wedge (1-\sigma)$is fuzzy semi–closed set in (Ω, ρ_s).

Proof:

Let(Ω, ρ_s) be a fuzzy semi–submaximal space and σ be a fuzzy set defined on Ω. By a known definition $[(1-\mathrm{cl_s}(\sigma)) \vee \sigma]$is a fuzzy semi–dense set in (Ω, ρ_s).

Conversely,assume $\mathrm{cl_s}[1-\mathrm{cl_s}(\sigma) \vee \sigma] \neq 1$. Then $1 - \mathrm{cl_s}[1-\mathrm{cl_s}(\sigma) \vee \sigma] \neq 0$ implies that $1 - [\mathrm{cl_s}\{1 - \mathrm{cl_s}(\sigma)\} \vee \mathrm{cl_s}(\sigma)] \neq 0$ it follows that $[(1 - \mathrm{cl_s}\{1 - \mathrm{cl_s}(\sigma)\}) \wedge (1-\mathrm{cl_s}(\sigma))] \neq 0$ gives $\mathrm{int_s cl_s}(\sigma) \wedge (1-\mathrm{cl_s}(\sigma)) \neq 0$. This shows that $\mathrm{int_s cl_s}(\sigma) \leq (1-\mathrm{cl_s}(\sigma))$. Thus, $\mathrm{int_s cl_s}(\sigma) \leq \mathrm{cl_s}(\sigma)$which gives a contradiction.

Hence it must be $\mathrm{cl_s}[(1 - \mathrm{cl_s}(\sigma)) \vee \sigma] = 1$ in (Ω, ρ_s). Since the space (Ω, ρ_s) is fuzzy semi–submaximal space$[(1 - \mathrm{cl_s}(\sigma))] \vee \sigma$. Therefore, $\mathrm{cl_s}(\sigma) \wedge (1-\sigma)$is the fuzzy semi–closed set.

Proposition-4.9

If$cl_s(\sigma)=1$, where σ is a fuzzy semi-dense set, then $cl_s(sint(\sigma)) = cl_s(\sigma)$ in a fuzzy semi-submaximal space (Ω, ρ_s).

Proof:

Let σ be a fuzzy semi–dense set in (Ω, ρ_s). Since (Ω, ρ_s) isa fuzzy semi–submaximal space such that $cl_s(\sigma)=1$ is fuzzy semi–open set in (Ω, ρ_s). It follows that, $\sigma \leq cl_s int_s(\sigma)$ in (Ω, ρ_s) implies that $cl_s(\sigma) \leq cl_s[cl_s int_s(\sigma)]$ and $cl_s(\sigma) \leq cl_s int_s(\sigma)$. Given that $cl_s int_s(\sigma) \leq cl_s(\sigma)$ Therefore, $cl_s(int_s(\sigma)) = cl_s(\sigma)$ in (Ω, ρ_s).

5. *Interrelation of Fuzzy Semi–Submaximal Spaces*

Proposition-5.1

If $cl_s\left(\overset{\infty}{\underset{i=1}{\wedge}}(\sigma_i)\right) = 1$, where σ_i isfuzzy semi–dense sets in a fuzzy semi–submaximal space (Ω, ρ_s), then(Ω, ρ_s) is fuzzy semi–Baire space.

Proof:

In fuzzy semi-submaximal space ifσ_i isfuzzy semi–dense sets,then σ_i ε ρ in (Ω, ρ_s). Since $\sigma_i$are fuzzy semi-submaximal space such that $cl_s(\sigma_i) = 1$ and $int_s(\sigma_i) = \sigma_i$ implies that $cl_s int_s(\sigma_i) = 1$. Then we have $1\text{-}cl_s int_s(\sigma_i) = 0$. By a lemma 1, gives that $int_s cl_s(1\text{-}\sigma_i) = 0$. Therefore,$1-\sigma_i$is fuzzy semi-nowhere dense sets in (Ω, ρ_s). Now $cl_s\left(\overset{\infty}{\underset{i=1}{\wedge}}(\sigma_i)\right) = 1$, implies that $1 - cl_s\left(\overset{\infty}{\underset{i=1}{\wedge}}(\sigma_i)\right) = 0$. Then by lemma 1, $int_s\left(\overset{\infty}{\underset{i=1}{\vee}}(1 - \sigma_i)\right) = 0$. Hence by the definition, it is proved that (Ω, ρ_s) is fuzzy semi–Baire space.

Proposition-5.2

Let (Ω, ρ) is fuzzy submaximal space, then (Ω, ρ) is fuzzy irresolvable and fuzzy semi-submaximal spaces in (Ω, ρ_s).

Proof:

Consider,the fuzzy set 1–σbe a fuzzy submaximal space,then 1–σ is fuzzy open set and $int_s(1-\sigma) = \sigma \neq 0$ in (Ω, ρ_s). Now by lemma 1, $cl_s(\sigma) = 1\text{-}int_s(1-\sigma) = 1 - (1 - \sigma) = \sigma \neq 1$ is showed to get fuzzy irresolvable space in (Ω, ρ_s). By hypothesis the fuzzy dense sets are fuzzy semi-dense set 1–σ with $int_s(1-\sigma) \neq 0$, $cl_s int_s(1-\sigma) = cl_s(1-\sigma) = 1$. Therefore, (Ω, ρ_s) gives a fuzzy semi-submaximal space.

Proposition-5.3

Afuzzy semi-topological space, if (Ω, ρ) is fuzzy almost resolvable and (Ω, ρ_s) is fuzzy semi–submaximal space, then (Ω, ρ_s) does not have a fuzzy semi-second category space.

Proof:

If (Ω, ρ) isafuzzy almost resolvable space, then, $\overset{\infty}{\underset{i=1}{\vee}}(\sigma_i) = 1$ where σ_i is such that $int(\sigma_i) = 0$ in (Ω, ρ). By hypothesis every fuzzy topological space is fuzzy semi-topological space. It is given that (Ω, ρ_s) is fuzzy semi–submaximal space then by the proposition 4.3$int_s(\sigma_i) = 0$ in(Ω, ρ_s) implies $int_s cl_s(\sigma_i) = 0$ in (Ω, ρ_s). It is provedσ_i is fuzzy semi-nowhere dense sets in (Ω, ρ_s). Therefore, $\vee_{i=1}^{\infty}(\sigma_i) = 1$ where $\sigma_i$is a fuzzy semi-first category space. Hence, (Ω, ρ_s) does not satisfy the fuzzy semi-second category space.

Proposition-5.4

To prove that in a fuzzy semi-topological space, if (Ω, ρ) is fuzzy submaximal space, then (Ω, ρ) is fuzzy semi–submaximal space in (Ω, ρ_s).

Proof:

If (Ω, ρ) is a fuzzy submaximal space, then every fuzzy dense set σ is a fuzzy open set in (Ω, ρ). By given hypothesis, every fuzzy dense set isfuzzy semi-dense set and since each fuzzy open set is also a fuzzy semi–open set in (Ω, ρ_s). Hence (Ω, ρ_s) is a fuzzy semi–submaximal space.

Remark:

From the above proposition it shows that every fuzzy submaximal space is fuzzy semi–submaximal space. However,the converse of fuzzy semi-submaximal space need not necessarily be a fuzzy submaximal space.

6. Applications

Fuzzy submaximal space is one of the fuzzy topological space. Fuzzy semi-submaximal space is the extension of fuzzy submaximal space. It is mostly used in many fields like Decision making, Engineering, Medical Diagnosis, Advanced Data Analysis and Robotics and Submaximal Fitness test. The fuzzy semi-submaximal space is weaker than the fuzzy submaximal space.

1. **Decision making:** It is employed to structure data that contains vague information, allowing for better decision making. It is useful in system analyzing the fuzzy semi-open or fuzzy semi-closed boundaries.
2. **Engineering:** Fuzzy topological structures used to manage processes with uncertainity such as industrial automations like temperature and pressure. It is also used in consumer electronics which include optimization in households like washing machine and air conditioners by the use of fuzzy sets, fuzzy semi-open and fuzzy semi-closed.
3. **Medical Diagnosis:** Fuzzy sets and their related fuzzy topological properties like fuzzy semi-open, fuzzy semi-interior, fuzzy pre-interior are used to symptoms such as high pain or mild pain.
4. **Advanced Data Analysis and Robotics:** It is used for classifying and structuring data where the

boundaries of fuzzy semi-open and fuzzy semi-closed properties are used.

5. **Submaximal fitness test:** Fuzzy semi-submaximal spaces and related fuzzy topological spaces are in submaximal fitness test to model the uncertainty, and ambiguity in physiological data (like heart rate ((HR), walk test). It is used to enhance the accuracy of fitness. Fuzzy decision support use input variables like heart rate, fatigue, and walk test. The walk test is taken by the minute walk test (like 3 min walk test (3MWT), 1 min walk test (1MWT)). It is also tested at the resting time of the heart rate after the walk test. It is mainly used for oxygen level during the walk test. The fuzzy submaximal fitness test was carried out by the age, height, weight, leg length of both male and female participants. It discusses how fast the HR increases when walking and how fast the HR decreases once stop walking. This fitness test tells about the health care of the human body.

7. *Conclusion*

Throughout this paper we discussed about that a fuzzy semi-topological space becomes fuzzy semi–submaximal space. The condition that was studied is fuzzy semi-nowhere dense set becomes fuzzy semi-open set is also discussed. It is developed that the fuzzy semi-submaximal space becomes semi-open and semi-closed sets in fuzzy. It was investigated that the interrelation between fuzzy semi–submaximal space with semi-nowhere dense sets, semi–dense sets, semi–Baire space, irresolvable and almost resolvable space in fuzzy was studied.It is also discussed given an example for fuzzy semi-submaximal space. Additionally it is discussed that fuzzy submaximal space is fuzzy semi-submaximal space.

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