The Bioscan
AN INTERNATIONAL QUARTERLY JOURNAL OF LIFE SCIENCES

21(1) S.I (1) 74-100, 2026 www.thebioscan.com

Double-hump soliton excitations in an Antiferromagnetic spin lattice

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https://doi.org/10.63001/tbs.2026.v21.i01 S.I(1).pp74-100

KEYWORDS
Antiferromagnetic spin system, Holstein- Primakoff transformation, coherent state ansatz, time-dependent variational principle, Sine-Cosine function method, double-hump soliton

Received on: 13-12-2025
Acceptedon: 06-02-2026

Published on: 13-02-2026

ABSTRACT

Using a modified Hamiltonian that takes into account both the bilinear and D- M interactions as well as the biquadratic anisotropic interaction, the dynamical theory of soliton excitation in a one-dimensional antiferromagnetic spin lattice is investigated. The two coupled nonlinear partial differential equations that govern the dynamics of the system are derived by introducing the Holstein-Primakoff transformation, the coherent state ansatz, and the time-dependent variational principle. The double- hump soliton propagation, the effect of interaction, and finally the inhomogenity parameters are examined by the Sine-Cosine function method.

1 Introduction

In recent decades, a large amount of work has been conducted in the classical and semi-classical limit on nonlinear spin excitations in terms of solitons and solitary waves in magnetic systems [1-14]. For a continuum version of the classical linear Heisen- berg chain, the classical approach yields general soliton solutions [15-17]. Since the bosonic representation of the spin operators in a quantum spin system allows for the systematic incorporation of quantum corrections, it turns out to be the best method for studying the solitary waves. The alternative coherent states method [18-21] fur- ther approximate the Hamiltonian to be biquadratic in boson operators and uses a severely truncated Holstein-Primakoff (H-P) expansion [22] for $S^{\pm}$. Finding a soli- tary wave profile that is identical to classical solutions by using Glauber's coherent state representation [23] and using long-wave and small amplitude approximations is known as semiclassical treatment.Soliton-like excitations in a spin chain with a biquadratic anisotropic exchange interaction have been studied in recent decades using the coherent state technique [24,25].

Significant dynamical models that exhibit fascinating nonlinear behaviours have also been used to Magnetic systems with various kinds of interactions. The D-M inter- action is essential to understanding the special properties of some magnetic materials. Dzyaloshinski discovered the D-M interaction, an antisymmetric superexchage inter- action between two spins in quantum antiferromagnetic systems [26]. Later, Moriya noted that these interactions arise spontaneously from the spin-orbit coupling under low symmetry and depend on the system's microscopic characteristics [27] . Lissouck and Nguenang investigated the solitary excitations in a one-dimensional AFM with D- M interaction Using the many scales approach and perturbation theory [28]. A model of the Heisenberg antiferromagnetic spin system with the D-M interaction [29], bilin- ear and biquadratic interaction [30] has been studied for a continuous approximation. An examination of how inhomogeneity affects the soliton under perturbation [31-35] shows that inhomogeneity causes soliton splitting, which in turn causes a disorder in the system. Using a semi-classical approach that combines the multiple scale method with a quasi-discreteness approximation and Glauber's coherent-state representation, the current authors [36] have recently examined the bright and dark intrinsic localized spin modes of the AFM spin system with bilinear, biquadratic, anisotropic and D-M interaction using the discrete approximation method. However, double-hump soliton excitations [37,38] in the afore mentioned system have not been documented in the literature as far as we are aware. Motivated by this, in this work we use the continuum approximation to study the double-hump soliton excitation. In this study, we use the Holstein-Primakoff transformation, coherent state ansatz, and perturbation technique to investigate the double-hump soliton propagation in the system. We investigate the nature of double-hump soliton propagation and the effects of inhomogenity in the case of an inhomogeneous AFM system using the Sine-Cosine perturbation method.

The structure of this paper is as follows. The model Hamiltonian and associated equation of motion are presented in section 2. Section 3 provides the Sine-Cosine function approach for solving the resulting equations. A model for the inhomogeneous AFM system is proposed in section 4, and the effects of inhomogeneity are examined in section 5. Finally, section 6 presents the conclusions.

2 Equations of motion based on the Model Hamiltonian

AFM spin system with bilinear, biquadratic, uniaxial anisotropic and D-M interactions are taken into consideration [36]. The Hamiltonian can be written as

$$\tilde{H} = \sum_j \Big[\tilde{J}\left(\vec{S}_j^{A}\cdot\vec{S}_j^{B} + \vec{S}_{j+1}^{A}\cdot\vec{S}_j^{B}\right) + \tilde{J}'\left(\left(\vec{S}_j^{A}\cdot\vec{S}_j^{B}\right)^2 + \left(\vec{S}_{j+1}^{A}\cdot\vec{S}_j^{B}\right)^2\right)$$
$$+\tilde{A}\left(\left(\vec{S}_j^{AZ}\right)^2 + \left(\vec{S}_j^{BZ}\right)^2\right) + \tilde{A}'\left(\left(\vec{S}_j^{AZ}\right)^4 + \left(\vec{S}_j^{BZ}\right)^4\right)\Big]$$
$$+\tilde{D}\vec{Z}\cdot\left(\vec{S}_j^{A}\times\vec{S}_j^{B} + \vec{S}_{j+1}^{A}\times\vec{S}_j^{B}\right). \qquad (1)$$

Here, the bilinear and biquadratic interaction coefficients in this instance are denoted by $\tilde{J}$ and $\tilde{J}'$ respectively. $\tilde{D}$ represents the D-M interaction parameter, while $\tilde{A}$ and $\tilde{A}'$ represent the single site uniaxial anisotropic energy resulting from the crystal field effect. $\vec{S}_j^{A}$ and $\vec{S}_j^{B}$ are the spin operators at the two interpenetrating sublattice sites, A and B.

The dimensionless form of the Hamiltonian is expressed by using $H = \frac{\tilde{H}}{\hbar^2 S^2}$; $J = \tilde{J}$; $J' = \tilde{J}'\hbar^2 S^2$; $A = \tilde{A}$; $A' = \tilde{A}'\hbar^2 S^2$ and $D = \tilde{D}$. Thus the spin Hamiltonian (1) has the following form :

$$H = \sum_j \Big[\frac{J}{2S^2}\Big(\hat{S}_j^{A+}\hat{S}_j^{B-} + \hat{S}_j^{A-}\hat{S}_j^{B+} + \hat{S}_{j+1}^{A+}\hat{S}_j^{B-} + \hat{S}_{j+1}^{A-}\hat{S}_j^{B+} + 2\hat{S}_j^{AZ}\hat{S}_j^{BZ}$$
$$+2\hat{S}_{j+1}^{AZ}\hat{S}_j^{BZ}\Big) + \frac{J'}{4S^4}\Big(\hat{S}_j^{A+}\hat{S}_j^{A+}\hat{S}_j^{B-}\hat{S}_j^{B-} + \hat{S}_j^{A-}\hat{S}_j^{A-}\hat{S}_j^{B+}\hat{S}_j^{B+}$$
$$+\hat{S}_{j+1}^{A+}\hat{S}_{j+1}^{A+}\hat{S}_j^{B-}\hat{S}_j^{B-} + \hat{S}_{j+1}^{A-}\hat{S}_{j+1}^{A-}\hat{S}_j^{B+}\hat{S}_j^{B+} + 2\hat{S}_j^{A+}\hat{S}_j^{A-}\hat{S}_j^{B+}\hat{S}_j^{B-}$$
$$+2\hat{S}_{j+1}^{A+}\hat{S}_{j+1}^{A-}\hat{S}_j^{B+}\hat{S}_j^{B-} + \hat{S}_j^{AZ}\hat{S}_j^{BZ}\hat{S}_j^{AZ}\hat{S}_j^{BZ} + \hat{S}_j^{A+}\hat{S}_j^{B-}\hat{S}_j^{AZ}\hat{S}_j^{BZ}$$
$$+\hat{S}_j^{A-}\hat{S}_j^{B+}\hat{S}_j^{AZ}\hat{S}_j^{BZ} + \hat{S}_{j+1}^{AZ}\hat{S}_j^{BZ}\hat{S}_{j+1}^{AZ}\hat{S}_j^{BZ} + \hat{S}_{j+1}^{A+}\hat{S}_j^{B-}\hat{S}_{j+1}^{AZ}\hat{S}_j^{BZ}$$
$$+\hat{S}_{j+1}^{A-}\hat{S}_j^{B+}\hat{S}_{j+1}^{AZ}\hat{S}_j^{BZ}\Big) + \frac{iD}{2S^2}\Big(\hat{S}_j^{A+}\hat{S}_j^{B-} + \hat{S}_j^{A-}\hat{S}_j^{B+} +$$
$$\hat{S}_{j+1}^{A+}\hat{S}_j^{B-} + \hat{S}_{j+1}^{A-}\hat{S}_j^{B+}\Big) + \frac{A}{S^2}\left(\left(\hat{S}_j^{AZ}\right)^2 + \left(\hat{S}_j^{BZ}\right)^2\right) + \frac{A'}{S^4}$$
$$\left(\left(\hat{S}_j^{AZ}\right)^4 + \left(\hat{S}_j^{BZ}\right)^4\right)\Big]. \qquad (2)$$

In the semi-classical limit, the soliton excitation in the 1D AFM system can be studied using the Holstein-Primakoff (H-P) representation of the spin operators [22]

$$\hat{S}_j^{A+} = \left[2S - a_j^\dagger a_j\right]^{\frac{1}{2}} a_j ;\ \hat{S}_j^{A-} = a_j^\dagger \left[2S - a_j^\dagger a_j\right]^{\frac{1}{2}} ;\ \hat{S}_j^{AZ} = S - a_j^\dagger a_j . \tag{3}$$

$$\hat{S}_j^{B+} = b_j^\dagger \left[2S - b_j^\dagger b_j\right]^{\frac{1}{2}} ;\ \hat{S}_j^{B-} = \left[2S - b_j^\dagger b_j\right]^{\frac{1}{2}} b_j ;\ \hat{S}_j^{BZ} = -S + b_j^\dagger b_j . \tag{4}$$

to obtain the bosonized Hamiltonian, which is the simplified form of the Hamiltonian (2).The bosonic operators $a_j(b_j)$ and $a_j^\dagger(b_j^\dagger)$ satisfy the classic commutation relation $[a_m, a_n^\dagger] = \delta_{mn}$, $[a_m, a_n] = [a_m^\dagger, a_n^\dagger] = 0$. Since the ground state excitation value of $a_j, a_j^\dagger$ is small in comparison of $2S$, the H-P transformation for spin operators can be extended in a power series with the dimensionless parameter $\varepsilon = \frac{1}{S}$ for large spins in the low temperature limit. The equation of motion can be found using Glauber's coherent state approach [23] and the time dependent variational principle [39,40]. The coherent state amplitudes that satisfy the relations $a_j|\psi(t)\rangle = \alpha_j|\psi(t)\rangle$, $b_j|\psi(t)\rangle = \beta_j|\psi(t)\rangle$ and $|0\rangle$ as the vacuum state of the boson system are denoted by the values α_j and β_j. The resulting equation of motion for the coherent state amplitudes α_j and β_j, can be written as follows:

$$\begin{aligned} i\hbar\frac{d\alpha_j}{dt} = {} & S^4\left[\left(2J - 4J' + 2A + 4A'\right)\alpha_j + \left(J - 2J' - iD\right)\left(\beta_j^\dagger + \beta_{j-1}^\dagger\right)\right] \\ & + S^3\left[\left(-J + 6J'\right)\left(\alpha_j\beta_j^\dagger\beta_j + \alpha_j\beta_{j-1}^\dagger\beta_{j-1}\right) + \left(2J' + A + 6A'\right)\right. \\ & 2\alpha_j^\dagger\alpha_j\alpha_j + \left(-\frac{1}{4}J + \frac{3}{2}J' - \frac{1}{4}iD\right)\left(\alpha_j\alpha_j\beta_j + \alpha_j\alpha_j\beta_{j-1}\right) + \\ & \left(-\frac{1}{4}J + \frac{3}{2}J' + \frac{1}{4}iD\right)\left(\beta_j^\dagger\beta_j^\dagger\beta_j + 2\alpha_j\alpha_j^\dagger\beta_j^\dagger + \beta_{j-1}^\dagger\beta_{j-1}^\dagger\beta_{j-1}\right. \\ & \left.\left. + 2\alpha_j\alpha_j^\dagger\beta_{j-1}^\dagger\right) + 2J'\left(\alpha_j^\dagger\beta_j^\dagger\beta_j^\dagger + \alpha_j^\dagger\beta_{j-1}^\dagger\beta_{j-1}^\dagger\right)\right], \end{aligned} \tag{5}$$

$$\begin{aligned} i\hbar\frac{d\beta_j}{dt} = {} & S^4\left[\left(2J - 4J' + 2A + 4A'\right)\beta_j + \left(J - 2J' - iD\right)\left(\alpha_j^\dagger + \alpha_{j+1}^\dagger\right)\right] \\ & + S^3\left[\left(-J + 6J'\right)\left(\alpha_j\alpha_j^\dagger\beta_j + \alpha_{j+1}\alpha_{j+1}^\dagger\beta_j\right) + \left(2J' + A + 6A'\right)\right. \\ & 2\beta_j\beta_j\beta_j^\dagger + \left(-\frac{1}{4}J + \frac{3}{2}J' - \frac{1}{4}iD\right)\left(\alpha_j\beta_j\beta_j + \alpha_{j+1}\beta_j\beta_j\right) + \\ & \left(-\frac{1}{4}J + \frac{3}{2}J' + \frac{1}{4}iD\right)\left(\alpha_j\alpha_j^\dagger\alpha_j^\dagger + 2\alpha_j^\dagger\beta_j\beta_j^\dagger + \alpha_{j+1}\alpha_{j+1}^\dagger\alpha_{j+1}^\dagger\right. \\ & \left.\left. + 2\alpha_{j+1}^\dagger\beta_j\beta_j^\dagger\right) + 2J'\left(\alpha_j^\dagger\alpha_j^\dagger\beta_j^\dagger + \alpha_{j+1}^\dagger\alpha_{j+1}^\dagger\beta_j^\dagger\right)\right]. \end{aligned} \tag{6}$$

The nonlinearity and discreteness of equations 5 and 6 make them difficult to solve exactly. The exact solution is thus obtained by using the continuum approximation. This is accomplished by substituting the functions $\alpha_j(t)$ and $\beta_j(t)$ with $\psi_1(x, t)$ and

$\psi_2(x, t)$ respectively. The following is the Taylor series expansion:

$$\alpha_{j\pm1}(t) = \psi_1(t) \pm \eta\psi_{1x} + \frac{1}{2!}\eta^2\psi_{1xx} \pm \frac{1}{3!}\eta^3\psi_{1xxx} + \frac{1}{4!}\eta^4\psi_{1xxxx} \pm \ldots\ldots \quad (7)$$

$$\beta_{j\pm1}(t) = \psi_2(t) \pm \eta\psi_{2x} + \frac{1}{2!}\eta^2\psi_{2xx} \pm \frac{1}{3!}\eta^3\psi_{2xxx} + \frac{1}{4!}\eta^4\psi_{2xxxx} \pm \ldots\ldots \quad (8)$$

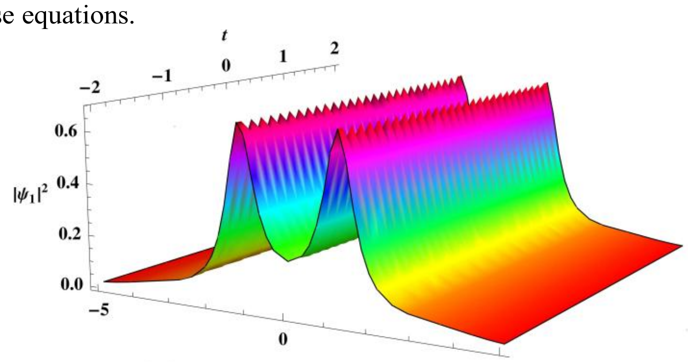
In this case η is a small dimensionless parameter. Equations 5 and 6 are solved using the continuum approximations 7 and 8 to obtain the following equations:

$$\begin{aligned} i\hbar\frac{d\psi_1}{dt} = {} & a_0\psi_1 + a_1P_0 + a_2P_1 + a_3iP_1 + a_4\ (P_2 + P_3 + P_4 \\ & -\eta\ (P_5 + P_6 + P_7) + \eta^2\ (P_8 + P_9 + P_{10})) + a_5\ i \\ & (-P_3 + P_4 - i\eta\ (-P_6 + P_7) + i\eta^2\ (-P_9 + P_{10})) \\ & + a_6\ (P_{11} - \eta P_{12} + \eta^2\ P_{13}), \end{aligned} \quad (9)$$

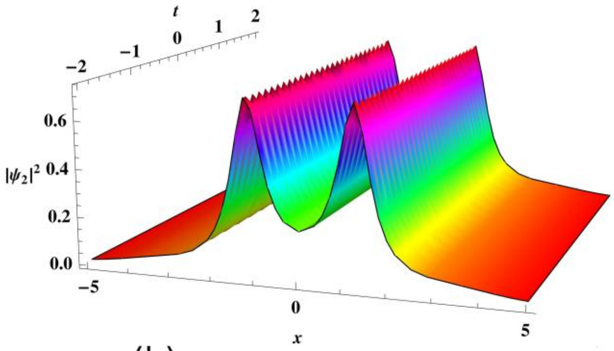
$$\begin{aligned} i\hbar\frac{d\psi_2}{dt} = {} & a_0\psi_2 + a_1Q_0 + a_2Q_1 + a_3iQ_1 + a_4\ (Q_2 + Q_3 + Q_4 \\ & +\eta\ (Q_5 + Q_6 + Q_7) + \eta^2\ (Q_8 + Q_9 + Q_{10})) \\ & - + a_5\ i\ (Q_3 + Q_4 - + i\eta\ (Q_6 + Q_7) + i\eta^2\ (Q_9 + Q_{10})) \\ & + a_6\ (Q_{11} + \eta Q_{12} + \eta^2\ Q_{13}), \end{aligned} \quad (10)$$

where $a_0 = 2S^4(A + 2A' + J - 2J')$; $a_1 = 2S^3(A + 6A' + 2J')$; $a_2 = S^4(-J + 2J')$; $a_3 = S^4D$; $a_4 = S^3(-J_1 + 6J_2^*)$; $a_5 = S^3D$; $a_6 = 4J'S^3$.

$P_0 = \psi_1^2\psi_1^*$; $P_1 = -2\psi_2^* + \eta\psi^*_{2x} - \frac{1}{2}\eta\, \psi_{2xx}$; $P_2 = 2\psi_1\psi_2\psi_2^*$; $P_3 = \frac{1}{2}\psi_1^2\psi_2$; $P_4 = \psi_1\psi_1^*\psi_2^* + \frac{1}{2}\psi_2\psi_2^{*2}$; $P_5 = \psi_1\psi_{2x}\psi_2^* + \psi_1\psi_2\psi^*_{2x}$; $P_6 = \frac{1}{4}\psi_1^2\psi_{2x}$; $P_7 = \frac{1}{4}\psi_{2x}\psi_2^{*2} + \frac{1}{2}\psi_1\psi_1^*\psi^*_{2x} + \frac{1}{2}\psi_2\psi_2^*\psi^*_{2x}$; $P_8 = \frac{1}{2}\psi_1\psi_{2xx}\psi_2^* + \psi_1\psi_{2x}\psi^*_{2x} + \frac{1}{2}\psi_1\psi_2\psi^*_{2xx}$; $P_9 = \frac{1}{8}\psi_1^2\psi_{2xx}$; $P_{10} = \frac{1}{8}\psi_{2xx}\psi_2^{*2} + \frac{1}{2}\psi_{2x}\psi_2^*\psi^*_{2x} + \frac{1}{4}\psi_2\psi^{*2}_{2x} + \frac{1}{4}\psi_1\psi_1^*\psi^*_{2xx} + \frac{1}{4}\psi_2\psi_2^*\psi^*_{2xx}$; $P_{11} = \psi_1^*\psi_2^{*2}$; $P_{12} = \psi_1^*\psi_2^*\psi^*_{2x}$; $P_{13} = \frac{1}{2}\psi_1^*\psi^{*2}_{2x} + \frac{1}{2}\psi_1^*\psi_2^*\psi^*_{2xx}$; $Q_0 = \psi_2^2\psi_2^*$; $Q_1 = -2\psi_1^* + \eta\psi^*_x - \frac{1}{2}\eta\psi^*_{xx}$; $Q_2 = 2\psi_1\psi_2\psi_1^*$; $Q_3 = \frac{1}{2}\psi_1\psi_2^2$; $Q_4 = \psi_2\psi_1^*\psi_2^* + \frac{1}{2}\psi_1\psi_1^{*2}$; $Q_5 = \psi_1\psi_2\psi_x^* + \psi_1\psi_x\psi_2^*$; $Q_6 = \frac{1}{4}\psi_2^2\psi_x$; $Q_7 = \frac{1}{4}\psi_x\psi_1^{*2} + \frac{1}{2}\psi_2\psi_2^*\psi_x^* + \frac{1}{2}\psi_1\psi_1^*\psi_x^*$; $Q_8 = \frac{1}{2}\psi_1\psi_2\psi^*_{xx} + \psi_1\psi_x\psi^*_x + \frac{1}{2}\psi_{xx}\psi_2\psi_1^*$; $Q_9 = \frac{1}{8}\psi_2^2\psi_{xx}$; $Q_{10} = \frac{1}{8}\psi_{xx}\psi_1^{*2} + \frac{1}{2}\psi_x\psi_x^*\psi_1^* + \frac{1}{4}\psi_1\psi_x^{*2} + \frac{1}{4}\psi_2\psi_2^*\psi^*_{xx} + \frac{1}{4}\psi_1\psi_1^*\psi^*_{xx}$; $Q_{11} = \psi_1^{*2}\psi_2^*$; $Q_{12} = \psi_1^*\psi_2^*\psi^*_x$; $Q_{13} = \frac{1}{2}\psi_2^*\psi_x^{*2} + \frac{1}{2}\psi_1^*\psi_2^*\psi^*_{xx}$. All the parameters are available in the first and second order derivative terms in equations (9) and (10). Also the higher order terms are very lengthy and while tested with mathematica software, their effect is found to be negligibly small. Hence we have excluded the higher order derivatives. Equations (9) and (10) are coupled nonlinear partial differential equations at the continuum limit. The perturbation method, discussed in the following section, is used to solve these equations.



(a)



(b)

Fig. 1 Double-hump soliton representing spin excitation with $c = -0.01, J = 12, J' = 6, A = 1, A' = 0.5, D = 1, \eta = 1$ and $S = 0.5$.

3 Double-hump Soliton Excitations

To obtain explicit travelling and solitary wave solutions to nonlinear evolution equations, numerous efficient methods have been used in the last few decades [41-48]. Our method of choice is the Sine-Cosine function method to generate Spin excitations of the solitary wave type. This approach involves writing $\psi_1 = E + iF$, $\psi_1^* = E - iF$, $\psi_2 = G + iH$, $\psi_2^* = G - iH$ in equations 9 and 10 . The wave variable $\xi = x - ct$ is then used to separate the real and imaginary parts and get the following equations:

$$-\hbar c F_\xi + a_0 E + a_1 \; E^3 + EF^2 \; + a_2 u_0 + a_3 u_1$$

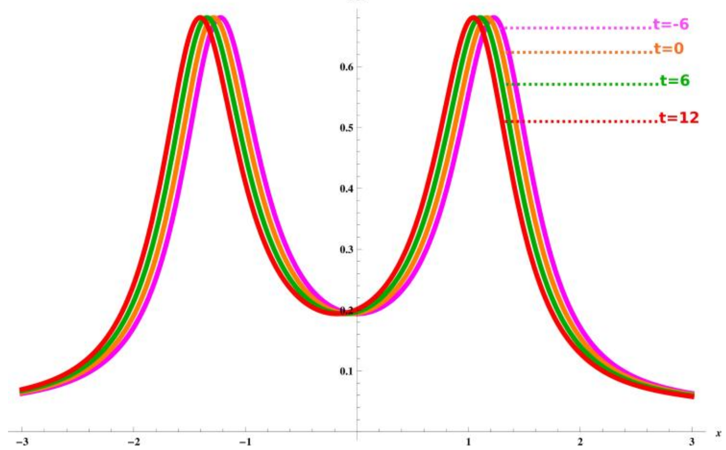


Fig. 2 Time variation of double-hump soliton representing spin excitation with $c = -0.01, J = 12, J' = 6, A = 1, A' = 0.5, D = 1, \eta = 1$ and $S = 0.5$.

$$+a_4u_2 + a_5u_3 + a_6u_4 = 0, \quad (11)$$

$$\hbar cE_\xi + a_0F + a_1 \left(E^2F + F^3\right) + a_2u_5 + a_3u_6 + a_4u_7 + a_5u_8 + a_6u_9 = 0, \quad (12)$$

$$-\hbar cH_\xi + a_0G + a_1 \left(G^3 + GH^2\right) + a_2u_{10} + a_3u_{11} + a_4u_{12} + a_5u_{13} + a_6u_{14} = 0 \quad (13)$$

and

$$\hbar cG_\xi + a_0H + a_1 \left(G^2H + H^3\right) + a_2u_{15} + a_3u_{16} + a_4u_{17} + a_5u_18 + a_6u_{19} = 0. \quad (14)$$

The expressions for $u_o, u_1, ..., u_{19} are in Appendix A$. The following forms can be used to solve equations (11 - 14):

$$E(x, t) = E(\xi) = \lambda_1 \cos^{\beta_1}(\mu\xi), \quad (15)$$

$$F(x, t) = F(\xi) = \lambda_2 \cos^{\beta_2}(\mu\xi), \quad (16)$$

$$G(x, t) = G(\xi) = \lambda_3 \cos^{\beta_3}(\mu\xi), \quad (17)$$

and

$$H(x, t) = H(\xi) = \lambda_4 \cos^{\beta_4}(\mu\xi). \quad (18)$$

Here λ_1, λ_2, λ_3 and λ_4 are the constant parameters. β $\beta_1, \beta_2, \beta_3$ and β_4 are determined by balancing the nonlinear term in equations (11 - 14) with the linear higher order derivative term, which yields $\beta_1 = \beta_2 = \beta_3 = \beta_4 = -1$. After that , a system of algebraic equation is obtained by using the values of $\beta_1, \beta_2, \beta_3$ and β_4 from equations (11 - 14). The following results can be obtained by solving the system of algebraic equations using symbolic computation:

$$\mu = \frac{-U_1 - \sqrt{U_1^2 - 4U_2U_3}}{2U_3}, \quad (19)$$

$$\lambda_1 = 2\sqrt{\frac{W_1}{W_2}}, \quad (20)$$

$$\lambda_2 = \lambda_4 = 0, \tag{21}$$

$$\lambda_3 = 2\sqrt{\frac{W_3}{W_4}}, \tag{22}$$

where

$U_1 = -16c\hbar^2 + 8\eta a_3 + 12\eta a_4 + 10\eta a_5,$
$U_2 = 16a_0 + 8a_1 - 16a_2 - 16a_3 + 8a_5$,
$U_3 = \eta^2 \quad -12a_2 + 12a_3 - 3a_4 + 8a_5 + 4a_6$,
$W_1 = -2c\hbar^2\mu - 2a_0 + 4a_2 + 2\eta\mu a_2 + \eta^2\mu^2 a_2 + 4a_3 + 2\eta\mu a_3 + \eta^2\mu^2 a_3,$
$W_2 = 8a_1 + 4a_4 - 6\eta\mu a_4 + 9\eta^2\mu^2 a_4 + 4a_5 - 6\eta\mu a_5 + 9\eta^2\mu^2 a_5,$
$W_3 = -2c\hbar^2\mu - 2a_0 + 4a_2 + 2\eta\mu a_2 + \eta^2\mu^2 a_2 + 4a_3 - 2\eta\mu a_3 + \eta^2\mu^2 a_3,$
$W_4 = 4a_4 - 6\eta\mu a_4 + 9\eta^2\mu^2 a_4 + 4a_5 - 6\eta\mu a_5 + 9\eta^2\mu^2 a_5.$

Thus the solutions to equations 9 and 10 are

$$\psi_1\ x,t = 2\sqrt{\frac{W_1}{W_2}} \times \sec h \quad -U_1 - \frac{\sqrt{U_1^2 - 4U_2U_3}}{2U_3} \times \quad x - ct \tag{23}$$

$$\psi_2\ x,t = 2\sqrt{\frac{W_3}{W_4}} \times \sec h \quad -U_1 - \frac{\sqrt{U_1^2 - 4U_2U_3}}{2U_3} \times \quad x - ct \tag{24}$$

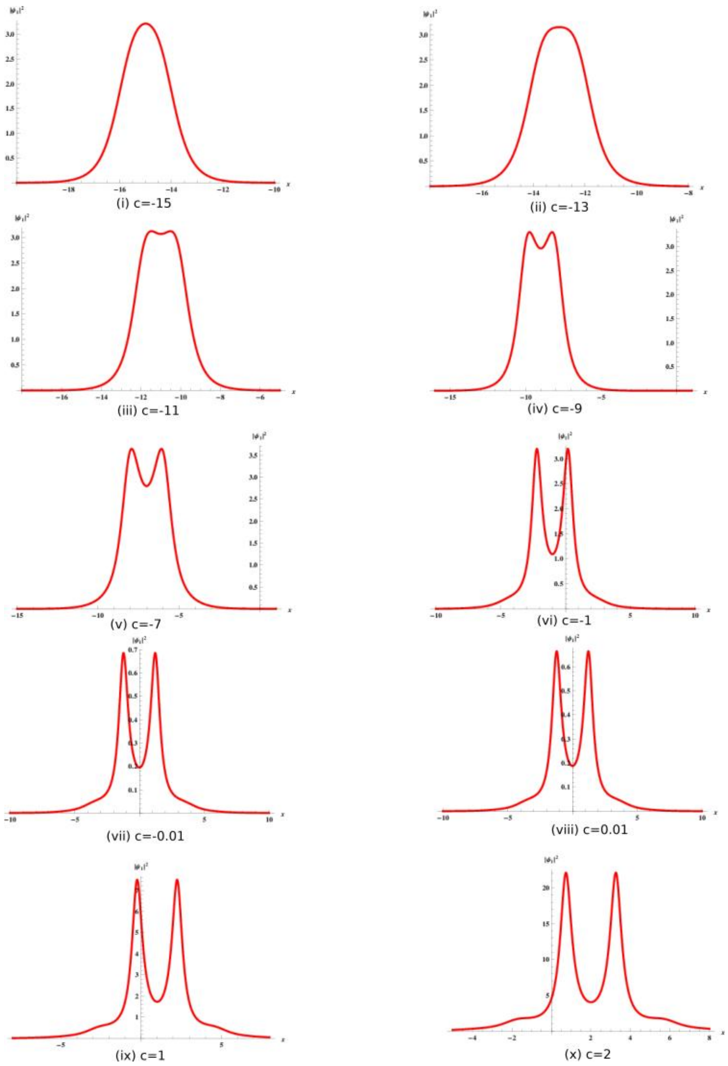


Fig. 3 Soliton profile for different values of speed(c).

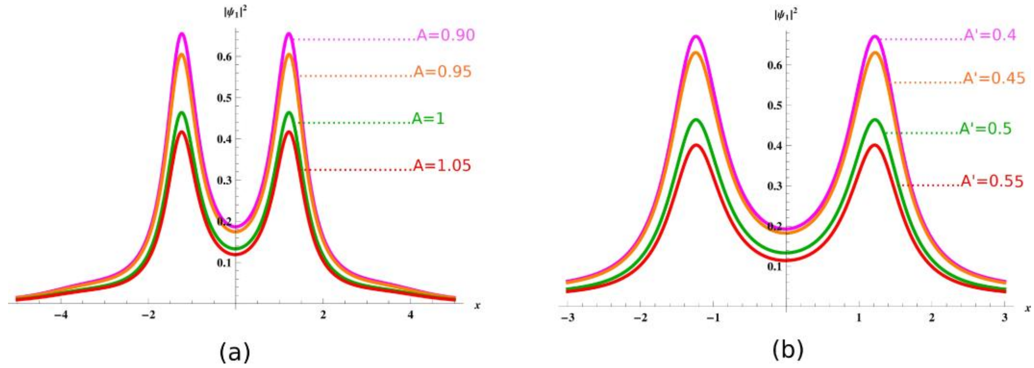


Fig. 4 Change in amplitude (a) for different values of A (b) for different values of A'.

Fig.1(a) graphically depicts equation 23 for the parametric values $c = -0.01, J = 12, J' = 6, A = 1, A' = 0.5, D = 1, \eta = 1$ and $S = 0.5$. Here, a stable double-hump soliton can be produced without any amplitude compression or broadening. Fig. 1(b) displays a similar profile that is produced by equation 24 . Fig.2 shows the motion of a solitary wave with a double-hump period of $t = 6$. The 2D plot of soliton for various speed(c) is shown in Fig.3. Here, we observe that when the speed increases, the single-hump soliton transforms into a multi-hump soliton. Consequently, a single-hump soliton splitting into multi-hump soliton due to speed fluctuation which signifies a lack of stability and a deviation from the soliton's typical self-reproducing behavior.

Fig.4(a) and 4(b) shows how the double-hump solitary density may alter when the anisotropy parameter changes. The amplitude and width of the double-hump solitary waves are observed to increase as the bilinear anisotropic energy parameter A and biquadratic anisotropic energy parameter A' decrease. As the speed of soliton is high for soliton with large amplitude, we conclude that AFM materials with low values of A and A' support fastest wave for long-distance signal transmission.

As shown in Fig.5, the amplitude and width of the solitary wave decrease as the bilinear interaction coefficients J and biquadratic interaction coefficients J' increase. when the J and J' are changed, height and width increase for particular values $J =$ 12.1 and $J' = 5.7$. It implies that the wave pulse amplitude and spatial extent both rise. This leads us to conclude that some materials permit energy transmission in the form of solitons, with J and J' values close to $J = 12.1$ and $J' = 5.7$. The density profiles dimension reduces when the D-M interaction parameter D rises, as seen in Fig.6. This suggests that the AFM with low D values promotes solitary wave propagation.

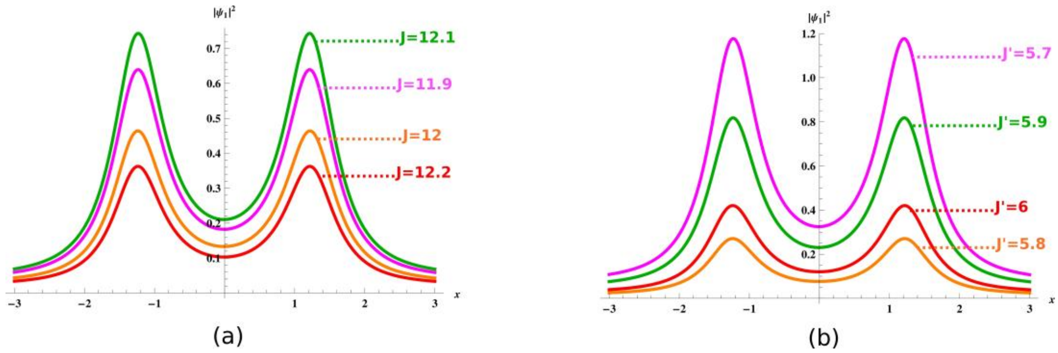


Fig. 5 Change in amplitude (a) for different values of J (b) for different values of J' .

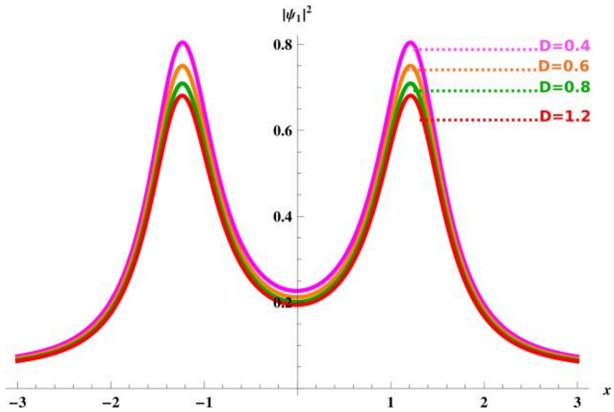


Fig. 6 Change in amplitude for different values of D.

4 The inhomogeneous model of AFM

For the AFM spin system with bilinear, biquadratic, anisotropic and D-M interaction, the impact of inhomogeneities is examined using the Hamiltonian

$$\tilde{H} = \sum_j f_j \Big[\tilde{J}\big(\vec{S}_j^A.\vec{S}_j^B + \vec{S}_{j+1}^A.\vec{S}_j^B\big) + \tilde{J}'\big((\vec{S}_i^A.\vec{S}_i^B)^2 + (\vec{S}_{j+1}^A.\vec{S}_j^B)^2\big) + \tilde{A}\big((\vec{S}_i^{AZ})^2 + (\vec{S}_i^{BZ})^2\big) + \tilde{A}'\big((\vec{S}_i^{AZ})^4 + (\vec{S}_i^{BZ})^4\big)\Big] + g_j \tilde{D}Z\big(\vec{S}_j^A \times \vec{S}_j^B + \vec{S}_{j+1}^A \times \vec{S}_j^B\big), \quad (25)$$

where the function f_j and g_j represent site dependent inhomogeneity. All other parameters are the same as that given in section 2. The dimensionless form of spin Hamiltonian (25) can be written as follows:

$$
\begin{aligned}
H = \sum_j \Big[& f_j \frac{J}{2S^2}\Big(\hat S^{A+}_j \hat S^{B-}_j + \hat S^{A-}_j \hat S^{B+}_j + \hat S^{A+}_{j+1}\hat S^{B-}_j + \hat S^{A-}_{j+1}\hat S^{B+}_j + 2\hat S^{AZ}_j \hat S^{BZ}_j \\
& + 2\hat S^{AZ}_{j+1}\hat S^{BZ}_j\Big) + \frac{J'}{4S^4}\Big(\hat S^{A+}_j \hat S^{A+}_j \hat S^{B-}_j \hat S^{B-}_j + \hat S^{A-}_j \hat S^{A-}_j \hat S^{B+}_j \hat S^{B+}_j \\
& + \hat S^{A+}_{j+1}\hat S^{A+}_{j+1}\hat S^{B-}_j \hat S^{B-}_j + \hat S^{A-}_{j+1}\hat S^{A-}_{j+1}\hat S^{B+}_j \hat S^{B+}_j + 2\hat S^{A+}_j \hat S^{A-}_j \hat S^{B+}_j \hat S^{B-}_j \\
& + 2\hat S^{A+}_{j+1}\hat S^{A-}_{j+1}\hat S^{B+}_j \hat S^{B-}_j + \hat S^{AZ}_j \hat S^{BZ}_j \hat S^{AZ}_j \hat S^{BZ}_j + \hat S^{A+}_j \hat S^{B-}_j \hat S^{AZ}_j \hat S^{BZ}_j \\
& + \hat S^{A-}_j \hat S^{B+}_j \hat S^{AZ}_j \hat S^{BZ}_j + \hat S^{AZ}_{j+1}\hat S^{BZ}_j \hat S^{AZ}_{j+1}\hat S^{BZ}_j + \hat S^{A+}_{j+1}\hat S^{B-}_j \hat S^{AZ}_{j+1}\hat S^{BZ}_j \\
& + \hat S^{A-}_{j+1}\hat S^{B+}_j \hat S^{AZ}_{j+1}\hat S^{BZ}_j\Big) + g_j \frac{iD}{2S^2}\Big(\hat S^{A+}_j \hat S^{B-}_j + \hat S^{A-}_j \hat S^{B+}_j \\
& + \hat S^{A+}_{j+1}\hat S^{B-}_j + \hat S^{A-}_{j+1}\hat S^{B+}_j\Big) + \frac{A}{S^2}\Big(\big(\hat S^{AZ}_j\big)^2 + \big(\hat S^{BZ}_j\big)^2\Big) \\
& + \frac{A'}{S^4}\Big(\big(\hat S^{AZ}_j\big)^4 + \big(\hat S^{BZ}_j\big)^4\Big)\Big]. \qquad (26)
\end{aligned}
$$

Using equations 3 and 4 in equation 26 , we construct the equation of motion for the coherent state amplitudes α_j and β_j as

$$
\begin{aligned}
i\hbar\frac{d\alpha_j}{dt} = {} & S^4\big(J - 2J'\big)\big(\beta^*_j f_j + \beta^*_{j-1} f_{j-1} + \alpha_j f_j + \alpha_j f_{j-1}\big) + S^4 \\
& - iD\big(\beta^*_j g_j + \beta^*_{j-1} g_{j-1}\big) + S^4\big(2A + 4A'\big)\alpha_j + \\
& S^3\Big(-\frac{1}{4}J + \frac{3}{2}J'\Big)\big(\alpha_j \alpha_j \beta_j f_j + \beta^*_j \beta^*_j \beta_j f_j + 2\alpha^*_j \alpha_j \beta^*_j f_j \\
& + \alpha_j \alpha_j \beta_{j-1} f_{j-1} + \beta_{j-1}\beta^*_{j-1}\beta^*_{j-1} f_{j-1} + 2\alpha_j \alpha^*_j \beta^*_{j-1} f_{j-1}\big) \\
& + S^3 \frac{1}{4} iD\big(-\alpha_j \alpha_j \beta_j g_j + \beta_j \beta^*_j \beta^*_j g_j + 2\alpha_j \alpha^*_j \beta^*_j g_j \\
& - \alpha_j \alpha_j \beta_{j-1} g_{j-1} + \beta_{j-1}\beta^*_{j-1}\beta^*_{j-1} g_{j-1} + 2\alpha_j \alpha^*_j \beta^*_{j-1} g_{j-1}\big) \\
& + S^3\big(-J + 6J'\big)\big(\alpha_j \beta^*_j \beta_j f_j + \alpha_j \beta^*_{j-1}\beta_{j-1} f_{j-1}\big) + S^3 \\
& J'\big(2\alpha^*_j \beta^*_j \beta^*_j f_j + 2\alpha^*_j \beta^*_{j-1}\beta^*_{j-1} f_{j-1} + 2\alpha_j \alpha_j \alpha^*_j f_j + \\
& 2\alpha_j \alpha_j \alpha^*_j f_{j-1}\big) + S^3\big(A + 6A'\big)\, 2\alpha_j \alpha_j \alpha^*_j\,, \qquad (27)
\end{aligned}
$$

$$i\hbar\frac{d\beta_j}{dt} = S^4 \; J - 2J' \;\; \alpha_j^* f_j + \alpha_{j+1}^* f_j \; + 2\beta_j f \; + S^4 \; - iD$$
$$\alpha_j^* g_j + \alpha_{j+1}^* g \;\; + S^4 \;\; 2A + 4A' \;\; \beta_j \; + S^3 \; - \frac{1}{4}$$
$$J + \frac{3}{2} J' \;\; \alpha_j \beta_j \beta_j f_j + \alpha_j \alpha_j^* \alpha_j^* f_j + 2\alpha_j^* \beta_j \beta_j^* f_j$$
$$+\alpha_{j+1}\beta_j\beta_j f_j \; + \alpha_{j+1}^* \alpha_{j+1}^* \;\; \alpha_{j+1} f_j \; + 2\alpha_{j+1}^* \;\; \beta_j \beta_j^* f_j$$
$$+S^3 \; \frac{1}{4} iD \;\;\; - \alpha_j \beta_j \beta_j g \; + \alpha_j \alpha_j^* \alpha_j^* g_j + 2\alpha_j^* \beta_j \beta_j^* g_j$$
$$-\alpha_{j+1}\beta_j\beta_j g_j + \alpha_{j+1}^* \alpha_{j+1}^* \alpha_{j+1} g_j + 2\alpha_{j+1}^* \;\; \beta_j \beta_j^* g_j$$
$$+S^3 \;\; -J + 6J' \;\;\; \alpha_j^* \alpha_j \beta_j f_j + \alpha_{j+1}^* \alpha_{j+1} \beta_j f_j \;\; +$$
$$S^3 J' \;\; 2\alpha_j^* \alpha_j^* \beta_j^* f_j + 2\alpha_{j+1}^* \;\; \alpha_{j+1}^* \;\; \beta_j^* f \; + 4\beta \; \beta \; \beta_j^* f$$
$$+S^3 \;\; A + 6A') \;\; 2\beta \; \beta \; \beta_j^* \; . \tag{28}$$

Using Taylor series expoansions 7 , 8 and

$$f_{j\pm1} = f \pm \eta f_x + \frac{1}{2!}\eta^2 f_{xx} \pm \frac{1}{3!}\eta^3 f_{xxx} + \frac{1}{4!}\eta^4 f_{xxxx} \pm \ldots\ldots.. \tag{29}$$

$$g_{j\pm1} = g \pm \eta_x + \frac{1}{2!}\eta^2 g_{xx} \pm \frac{1}{3!}\eta^3 g_{xxx} + \frac{1}{4!}\eta^4 g_{xxxx} \pm \ldots\ldots.. \tag{30}$$

in equations 27 and 28 yields the equations of motion for the inhomogeneous AFM in the continuum limit:

$$i\hbar\frac{d\psi_1}{dt} = b_0\psi_1 + b_1 P_0 + b_2 \; f R_1 + f R_2 + f_{xx} R_3 \;\; + b_3 \;\; i \;\; g P_1$$
$$+g_x R_4 + g_{xx} R_5 \;\; + b_4 \, f \;\; P_2 + P_3 + P_4 \;\; - \eta \;\; P_5 +$$
$$P_6 + P_7 \;\; + \eta^2 \;\; P_8 + P_9 + P_{10} \;\;\; + \;\; f_x \;\; \eta \;\; R_6 + R_7$$
$$+R_8 + R_9 \;\; - \eta^2 \;\; R_{10} + R_{11} + R_{12} \;\;\;\; + f_{xx} \;\; \eta^2$$
$$R_{13} + R_{14} + R_{15} \;\;\; + b_5 \; ig \;\;\; - P_3 + P_4 \; - \eta$$

$$-P_6 + P_7 + \eta^2 -P_9 + P_{10} + ig_x \eta R_7 - R_8$$
$$-R_9 + \eta^2 R_{11} + R_{12} + ig_{xx} \eta^2 R_{13} + R_{15}$$
$$+b_6 f P_0 + P_{11} - \eta P_{12} + \eta^2 P_{13} + f_x$$
$$-\eta R_{16} + \eta^2 R_{17} + f_{xx} \eta^2 R_{18} \quad (31)$$

and

$$i\hbar\frac{d\psi_2}{dt} = b_0\psi_2 + b_1 Q_0 + b_2 f\theta_1 + b_3 igQ_1 + b_4 f\ Q_2 + Q_3 + Q_4$$
$$+\eta\ Q_5 + Q_6 + Q_7 + +\eta^2\ Q_8 + Q_9 + Q_{10} + b_5 g$$
$$-i\ Q_3 + Q_4 + i\eta\ Q_6 + Q_7 + i\eta^2 - Q_9 + Q_{10}$$
$$+b_6 f\ Q_0 + Q_{11} + \eta Q_{12} + \eta^2\ Q_{13} \quad (32)$$

where $b_0 = 2S^4(A + 2A')$; $b_1 = 2S^3(A + 6A')$; $b_2 = S^4(-J + 2J')$; $b_3 = S^4 D$; $b_4 = S^3(-J + 6J')$; $b_5 = S^3 D$; $b_6 = J4S'^3$. $R_1 = -2\psi - 2\psi + 2\eta\psi^* \frac{1}{2}{}^2 \psi^*_{2xx} -$ $R_2 = \eta\ \psi_1 + \psi^*_2 - \eta^2 \psi^*_{2x}$; $R_3 = -\frac{1}{2}\eta^2\ \psi_1 + \psi^*_2$; $R_4 = \eta\psi^*_2 - \eta^2\psi^*_{2x}$; $R_5 = -\frac{1}{2}\eta^2\psi^*_2$; $R_6 = \psi_1\psi_2\psi^*_2$; $R_7 = \frac{1}{4}\psi_1^2\psi_2$; $R_8 = \frac{1}{2}\psi_1\psi_1^*\psi_2^*$; $R_9 = \frac{1}{4}\psi_2\psi_2^{*2}$; $R_{10} = \psi_1\psi_{2x}\psi_2^* + \psi_1\psi_2\psi^*_{2x}$; $R_{11} = \frac{1}{4}\psi_1^2\psi_{2x}$; $R_{12} = \frac{1}{2}\psi_1\psi_1^*\psi^*_{2x} + \frac{1}{4}\psi_{2x}\psi_2^{*2} + \frac{1}{2}\psi_2\psi_2^*\psi^*_{2x}$; $R_{13} = \frac{1}{8}\psi_1^2\psi_2$; $R_{14} = \frac{1}{2}\psi_1\psi_2\psi_2^*$; $R_{15} = \frac{1}{4}\psi_1\psi_1^*\psi_2^* + \frac{1}{8}\psi_2\psi_2^{*2}$; $R_{16} = \frac{1}{2}\psi_1^2\psi_1^* + \frac{1}{2}\psi_1^*\psi_2^{*2}$; $R_{17} = \psi^*\psi^*\psi^*_{2x}$; $R_{18} = \frac{1}{4}\psi_1^2\psi_1^* + \frac{1}{4}\psi^*\psi^*_2{}^2$; and $\theta_1 = -2\psi - 2\psi^*_1 + \eta\psi^*_x - \frac{1}{2}\eta^2\psi^*_{xx}$. Section 2 contains the expressions for $P_0, P_1, ..., P_{13}$ and $Q_0, Q_1, ..., Q_{13}$. In equations 31 and 32 , all the parameters are available in the first and second order derivative terms. The higher order terms are very lengthy and while tested with mathematica software their effect is found to be negligibly small. Hence we have excluded the higher order derivatives.

Therefore, again we obtain a set of coupled nonlinear partial differential equations from equations 31 and 32 . These equations represent the dynamics of the inhomogeneous AFM spin chain with bilinear, biquadratic, anisotropic and D-M interaction.

Equations 31 and 32 are solved using the Sine-Cosine function approach to examine the effect of inhomogeneity in soliton excitation. The following equations can be obtained by separating the real and imaginary components of equations 31 and 32 as in section 3 and using the wave variable $\xi = x - ct$:

$$\hbar cF_\xi + b_0E + b_1\ E^3 + EF^2 + b_2\ fv_0 + f_xv_1 + f_{xx}v_2 +$$
$$b_3\ gu_1 + g_xv_3 + g_{xx}v_4 + b_4\ fu_2 + f_xv_5 + f_{xx}v_6 +$$
$$b_5\ gu_3 + g_xv_7 + g_{xx}v_8 + b_6\ fv_9 + f_xv_10 + f_{xx}v_{11} = 0, \quad (33)$$

$$\hbar c E_\xi + b_0 F + b_1\ E^2F + F^3\ + b_2\ fv_{12} + f_x v_{13} + f_{xx} v_{14}\ + b_3\ gu_6 + g_x v_{15} + g_{xx} v_{16}\ + b_4\ fu_7 + f_x v_{17} + f_{xx} v_{18} + b_5\ gu_8 + g_x v_{19} + g_{xx} v_{20}\ + b_6\ fv_21 + f_x v_{22} + f_{xx} v_{23}\ = 0, \quad (34)$$

$$-\hbar c H_\xi + b_0 G + b_1\ G^3 + GH^2\ + b_2 fv_{24} + b_3 gu_{11} + b_4 fu_{12} + b_5 gu_{13} + b_6 fv_{25} = 0, \quad (35)$$

and

$$\hbar c G_\xi + b_0 H + b_1\ G^2 H + H^3\ + b_2 fv_{26} + b_3 gu_{16} + b_4 fu_{17} + b_5 gu_1 8 + b_6 fv_{27} = 0. \quad (36)$$

Appendix A contains the expressions for $u_0, u_1, \ldots, u_{18}$ and appendix B lists $v_0, v_1, \ldots, v_{27}$.

Following the steps examined in section 3, we get,

$$\mu_1 = -M_1 - \frac{\sqrt{M_1^2 - 4M_2M_3}}{2M_2}, \quad (37)$$

$$\lambda_5 = \frac{2\sqrt{A_1}}{\sqrt{A_2}}, \quad (38)$$

$$\lambda_6 = \lambda_8 = 0, \quad (39)$$

$$\lambda_7 = -\frac{2\sqrt{A_3}}{\sqrt{A_4}}. \quad (40)$$

Solutions of equations 31 and 32 are

$$\psi_1\ x, t) = \frac{2\sqrt{A_1}}{\sqrt{A_2}} \times \sec h\ \ -M_1 - \frac{\sqrt{M_1^2 - 4M_2M_3}}{2M_2} \times\ x - ct\ , \quad (41)$$

and

$$\psi_2\ x, t) = \frac{-2\sqrt{A_3}}{\sqrt{A_4}} \times \sec h\ \ -M_1 - \frac{\sqrt{M_1^2 - 4M_2M_3}}{2M_2} \times\ x - ct\ , \quad (42)$$

where

$M_1 = 8c\hbar^2 + 28f\eta b_4 + 8g\eta b_5 + 4\eta^2 b_2 f_x + 14\eta^2 b_4 f_x - 4\eta^2 b_6 f_x + 4\eta^2 b_3 g_x - 4\eta^2 b_5 g_x,$

$M_2 = -4f\eta^2 b_2 - 4g\eta^2 b_3 + 28f\eta^2 b_4 + 10g\eta^2 b_5 + 8f\eta^2 b_6,$

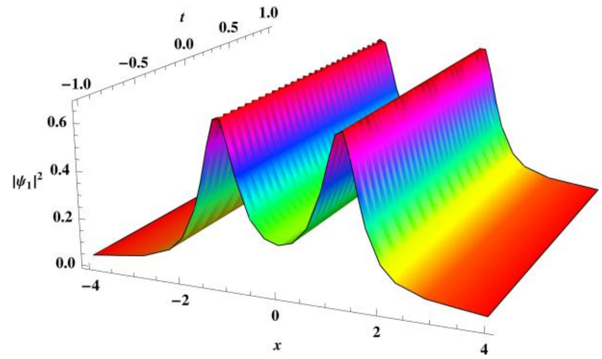


Fig. 7 Unperturbed double-hump soliton representing spin excitation with $c = -0.01, J = 12, J' = 6, A = 1, A' = 0.5, D = 1, \eta = 1, S = 0.5, f(x) = 1$ and $g(x) = 1$.

$M_3 = 8b_0 + 8b_1 - 32fb_2 - 16gb_3 + 32fb_4 + 8gb_5 + 16fb_6 + 8\eta b_2 f_x + 8\eta b_4 f_x - 4\eta b_6 f_x - 4\eta^2 b_2 f_{xx} + 4\eta^2 b_4 f_{xx} + b_6 f_{xx} + \eta^2 b_6 f_{xx} + 4\eta b_3 g_x - 2\eta b_5 g_x - 2\eta^2 b_3 g_{xx} + \eta^2 b_5 g_{xx},$

$A_1 = -2c\hbar^2\mu - 2b_0 + 8fb_2 + 2f\eta\mu b_2 + f\eta^2\mu^2 b_2 + 4gb_3 + 2g\eta\mu b_3 + g\eta^2\mu^2 b_3 - 2\eta b_2 f_x + \eta^2 b_2 f_{xx},$

$A_2 = 8b_1 + 4fb_4 - 6f\eta\mu b_4 + 9f\eta^2\mu^2 b_4 + 4gb_5 - 6g\eta\mu b_5 + 9g\eta^2\mu^2 b_5 + 8fb_6 - 4\eta b_6 f_x + 2\eta^2 b_6 f_{xx},$

$A_3 = -2c\hbar^2\mu - 2b_0 + 8fb_2 - 2f\eta\mu b_4 + f\eta^2\mu^2 b_2 + 4gb_3 - 2g\eta\mu b_3 + g\eta^2\mu^2 b_3 - 2\eta b_2 g_x + 2\eta^2\mu b_2 f_x + \eta^2 b_2 f_{xx} - 2\eta b_3 g_x + 2\eta^2\mu b_3 g_x + \eta^2 b_3 g_{xx},$ and

$A_4 = 8b_1 + 4fb_4 - 6f\eta\mu b_4 + 9f\eta^2\mu^2 b_4 + 4gb_5 - 6g\eta\mu b_5 + 9g\eta^2\mu^2 b_5 + 8fb_6 + 2\eta b_4 f_x - 6\eta^2\mu b_4 f_x + \eta^2 b_4 f_{xx} - 2\eta b_5 g_x + 6\eta^2\mu b_5 g_x + \eta^2 b_5 g_{xx}.$

5 Effect of inhomogeneity in soliton excitation

In many physical systems, the analytic soliton solutions are useful for explaining the nonlinear wave phenomena when inhomogeneities are present. It is anticipated that the coefficients of the equations in the inhomogeneous system will rely on the derivatives of the function characterizing the inhomogeneity. The site dependency or inhomogeneity of the coupling between spins is the source of the inhomogeneity functions $f(x)$ and $g(x)$ from the aforementioned solutions (41) and (42). The system produces the homogeneous Heisenberg AFM spin chain if the functions $f(x) = 1$ and $g(x) = 1$, as illustrated visually in Fig.7. For the parameters $c = -0.01, J = 12, J' = 6, A = 1, A' = 0.5, D = 1, \eta = 1, S = 0.5, f(x) = 1$ and $g(x) = 1$, it shows an double-hump soliton.

We have chosen cubic, biquadratic, periodic and localized types of inhomogeneities [49] to investigate whether the existence of these inhomogeneities alters soliton propagation in any way. Initially, we will examine cubic inhomogeneity of

the form $f(x) = 1 + Q_1x^3 + Q_2x^2$ and $g(x) = 1 + Q_3x^3 + Q_4x^2$, whose solution is shown in Fig.8. Plotting shows that the double-hump soliton stays steady for values $Q_1 < -0.009$, $Q_2 < -0.0005$, $Q_3 < -0.003$ and $Q_4 < -0.01$. In addition, we can see a disturbance in the tail region if we raise the values of Q_1, Q_2, Q_3 and Q_4. $f(x) = 1+Q_5x^4 +Q_6x^2$ and $g(x) = 1+Q_7x^4 +Q_8x^2$ are used to study the biquadratic inhomogeneity. Fig.9 demonstrates that the double-hump soliton maintains its shape when $Q_5 < -0.00149$, $Q_6 < -0.00125$, $Q_7 < -0.0015$ and $Q_8 < -0.001$. Above this value, a slight projection in the tail area is visible. In Fig.10, the periodic inhomogeneity $f(x) = 1 + Q_9 \sin x$ and $g(x) = 1 + Q_{10} \sin x$ is displayed. The double-hump soliton's hump exhibits a periodic variation for the values $Q_9=0.87$ and $Q_{10}=0.9$. For the functions $f(x) = 1 + Q_{11} \coth x$ and $g(x) = 1 + Q_{12} \coth x$, we can observe
a distortion in the localized region for the values $Q_{11}=1\times 10^{-12}$ and $Q_{12}=1\times 10^{-6}$ and the localized inhomogeneity is depicted in Fig.11. Function ψ_2 is excluded from this discussion as we have obtained similar findings. A tabulation of these results are provided in Table 1 and Table 2 for convenience.

Table 1 Limiting values with different types of inhomogeneities

Type of Inhomogeneity	Limiting Values
Without Inhomogeneity	–
Cubic	$Q_1 = -0.009$, $Q_2 = -0.0005$, $Q_3 = -0.003$, $Q_4 = -0.01$
Biquadratic	$Q_5=-0.00149$, $Q_6=-0.00125$, $Q_7=-0.0015$, $Q_8=-0.001$
Periodic	$Q_9=0.87$, $Q_{10}=0.9$
Localized	$Q_{11}=1\times 10^{-12}$, $Q_{12}=1\times 10^{-6}$

Table 2 The double-hump solitons dynamic behaviour with different types of inhomogeneities

Type of Inhomogeneity	Dynamic Behaviours
Without Inhomogeneity	Stable Propagation
Cubic	Disturbance in the tail region
Biquadratic	Projection in the tail area
Periodic	Hump exhibits a periodic variation
Localized	Distortion in the localized region

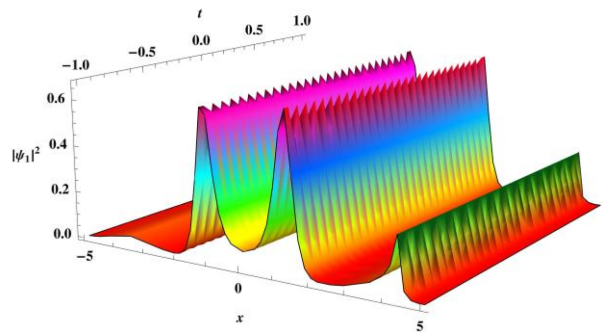


Fig. 8 Soliton evolution with cubic inhomogeneity for Q_1=−0.009, Q_2=−0.0005, Q_3=−0.003 and Q_4=−0.01.

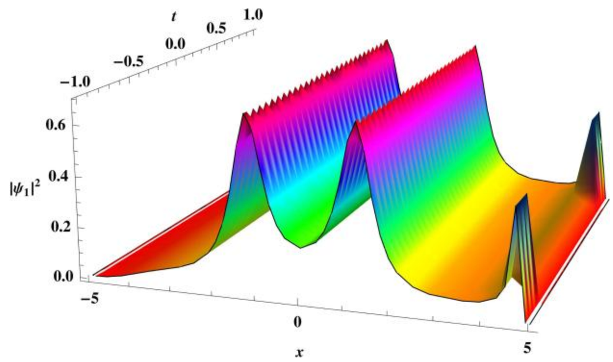


Fig. 9 Soliton evolution with biquadratic inhomogeneity for Q_5=−0.00149, Q_6=−0.00125, Q_7=−0.0015 and Q_8=−0.001

6 Conclusion

We study double-hump soliton excitations in an antiferromagnetic spin system with biquadratic and D-M interactions in addition to anisotropy and exchange energy. Using the coherent state ansatz, the time-dependent variational principle, and the Holstein-Primakoff transformation, it is discovered that the dynamics are governed by a set of two coupled nonlinear differential difference equations. Once the continumm equation of motion have been obtained using the long wave length approximation, they are solved using the Sine-Cosine perturbation technique, and the results are shown graphically. Our systems interaction parameters have been analyzed, and the results

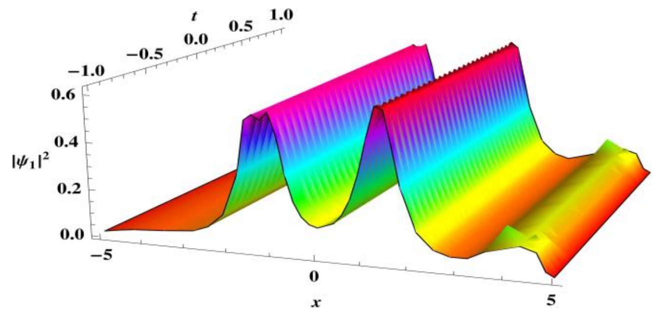


Fig. 10 Soliton evolution with periodic inhomogeneity for Q_9=0.87 and Q_{10}=0.9.

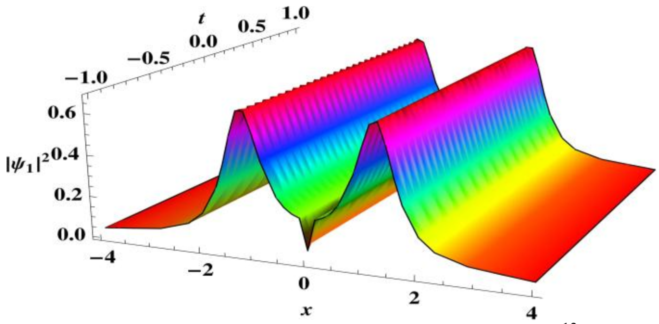


Fig. 11 Soliton evolution with localized inhomogeneity for $Q_{11}=1\times10^{-12}$and $Q_{12}=1\times10^{-6}$.

are shown graphically. Without any amplitude compression or broadening, the double-hump solitary wave is observed to be developing steadily. Additionally, using the same perturbation technique, a model for an inhomogeneous AFM system is built and the impact of inhomogeneity is examined. We study how soliton propagation behaves in an inhomogeneous AFM system for different kinds of nonlinear inhomogeneities. The perturbation analysis results indicates that when the quantity of inhomogeneity surpasses a limiting threshold, the double-hump soliton splits and the tail fluctuates. This suggests that soliton propagation is unstable, which impacts the system's capacity to operate normally.

Declarations

- Funding

The authors did not receive support from any organization for the submitted work.

- Conflict of interest

The authors (Sheneiga J and Latha M.M) certify that they have No affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affilations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

Appendix A

$u_0, u_1, \ldots, u_{19}$ of equation (11) - (14) are:

$u_0 = -2G + \eta G_x - \frac{1}{2}\eta^2 G_{xx};$

$u_1 = -2H + \eta H_x - \frac{1}{2}\eta^2 H_{xx};$

$u_2 = \frac{3}{2}E^2G + \frac{1}{2}F^2G - EFH + 2EG^2 + 2EH^2 + \frac{G^3}{2} + \frac{GH^2}{2} - \eta\left(\frac{3}{4}E^2G_x + \frac{1}{4}F^2G_x - \frac{1}{2}EFH_x + 2EGG_x + 2EHH_x + \frac{3}{4}G^2G_x + \frac{1}{4}H^2G_x + \frac{1}{2}GHH_x\right) + \eta^2\left(\frac{3}{8}E^2G_{xx} + \frac{1}{8}F^2G_{xx} - \frac{1}{4}EFH_{xx} + EGG_{xx} + EHH_{xx} + EG_x^2 + EH_x^2 + \frac{3}{8}G^2G_{xx} + \frac{1}{8}H^2G_{xx} + \frac{1}{4}GHH_{xx} + \frac{3}{4}GG_x^2 + \frac{1}{4}GH_x^2 + \frac{1}{2}HG_xH_x\right);$

$u_3 = EFG + \frac{3}{2}E^2H + \frac{1}{2}F^2H + \frac{1}{2}G^2H + \frac{1}{2}H^3 - \eta\left(\frac{1}{2}EFG_x + \frac{3}{4}E^2H_x + \frac{1}{4}F^2H_x + \frac{1}{2}GHG_x + \frac{1}{4}G^2H_x + \frac{3}{4}H^2H_x\right) + \eta^2\left(\frac{1}{4}EFG_{xx} + \frac{3}{8}E^2H_{xx} + \frac{1}{8}F^2H_{xx} + \frac{1}{4}GHG_{xx} + \frac{1}{8}G^2H_{xx} + \frac{3}{8}H^2H_{xx} + \frac{1}{4}HG_x^2 + \frac{3}{4}HH_x^2 + \frac{1}{2}GG_xH_x\right);$

$u_4 = EG^2 - 2FGH - EH^2 - \eta\left(EGG_x - FHG_x - FGH_x - EHH_x\right) + \eta^2\left(\frac{1}{2}EG_x^2 - FG_xH_x - \frac{1}{2}EH_x^2 + \frac{1}{2}EGG_{xx} - \frac{1}{2}FHG_{xx} - \frac{1}{2}FGH_{xx} - \frac{1}{2}EHH_{xx}\right);$

$u_5 = 2H - \eta H_x + \frac{1}{2}\eta^2 H_{xx};$

$u_6 = -2G + \eta G_x - \frac{1}{2}\eta^2 G_{xx};$

$u_7 = EFG - \frac{1}{2}E^2H - \frac{3}{2}F^2H + 2FG^2 + 2FH^2 - \frac{1}{2}G^2H - \frac{1}{2}H^3 - \eta\left(\frac{1}{2}EFG_x - \frac{1}{4}E^2H_x - \frac{3}{4}F^2H_x + 2FGG_x + 2FHH_x - \frac{1}{2}GHG_x - \frac{1}{4}G^2H_x - \frac{3}{4}H^2H_x\right) + \eta^2\left(\frac{1}{4}EFG_{xx} - \frac{1}{8}E^2H_{xx} - \frac{3}{8}F^2H_{xx} + FGG_{xx} + FHH_{xx} + FG_x^2 + FH_x^2 - \frac{1}{4}GHG_{xx} - \frac{1}{8}G^2H_{xx} - \frac{3}{8}H^2H_{xx} - \frac{1}{4}HG_x^2 - \frac{3}{4}HH_x^2 - \frac{1}{2}GG_xH_x\right);$

$u_8 = \frac{1}{2}E^2G + \frac{3}{2}F^2G + EFH + \frac{1}{2}G^3 + \frac{1}{2}GH^2 - \eta\left(\frac{1}{4}E^2G_x + \frac{3}{4}F^2G_x + \frac{1}{2}EFH_x + \frac{3}{4}G^2G_x + \frac{1}{4}H^2G_x + \frac{1}{2}GHH_x\right) + \eta^2\left(\frac{1}{8}E^2G_{xx} + \frac{3}{8}F^2G_{xx} + \frac{1}{4}EFH_{xx} + \frac{3}{8}G^2G_{xx} + \frac{1}{8}H^2G_{xx} + \frac{1}{4}GHH_{xx} + \frac{3}{4}GG_x^2 + \frac{1}{4}GH_x^2 + \frac{1}{2}HG_xH_x\right);$

$u_9 = -FG^2 - 2EGH + FH^2 - \eta\left(-FGG_x - EHG_x - EGH_x + FHH_x\right) + \eta^2\left(-\frac{1}{2}FG_x^2 - EG_xH_x + \frac{1}{2}FH_x^2 - \frac{1}{2}FGG_{xx} - \frac{1}{2}EGH_{xx} + \frac{1}{2}FHH_{xx}\right);$

$u_{10} = -2E - \eta E_x - \frac{1}{2}\eta^2 E_{xx};$

$u_{11} = -2F - \eta F_x - \frac{1}{2}\eta^2 F_{xx};$

$u_{12} = \frac{3}{2}EG^2 + \frac{1}{2}EH^2 - FGH + 2E^2G + 2F^2G + \frac{1}{2}E^3 + \frac{1}{2}EF^2 - \eta\left(\frac{3}{4}G^2E_x + \frac{1}{4}H^2E_x - \frac{1}{2}GHF_x + 2EGE_x + 2FGF_x + \frac{3}{4}E^2E_x + \frac{1}{4}F^2E_x + \frac{1}{2}EFF_x\right) + \eta^2\left(\frac{3}{8}G^2E_{xx} + \frac{1}{8}H^2E_{xx} - \frac{1}{4}GHF_{xx} + EGE_{xx} + FGF_{xx} + GE_x^2 + GF_x^2 + \frac{3}{8}E^2E_{xx} + \frac{1}{8}F^2E_{xx} + \frac{1}{4}EFF_{xx} + \frac{3}{4}EE_x^2 + \frac{1}{4}EF_x^2 + \frac{1}{2}FF_xE_x\right);$

$u_{13} = EGH + \frac{3}{2}FG^2 + \frac{1}{2}FH^2 + \frac{1}{2}E^2F + \frac{1}{2}F^3 + \eta\left(\frac{1}{2}GHE_x + \frac{3}{4}G^2F_x + \frac{1}{4}H^2F_x + \frac{1}{2}EFE_x + \frac{1}{4}E^2F_x + \frac{3}{4}F^2F_x\right) + \eta^2\left(\frac{1}{4}GHE_{xx} + \frac{3}{8}G^2F_{xx} + \frac{1}{8}H^2F_{xx} + \frac{1}{4}EFE_{xx} + \frac{1}{8}E^2F_{xx} + \frac{3}{8}F^2F_{xx} + \frac{1}{4}FE_x^2 + \frac{3}{4}FF_x^2 + \frac{1}{2}EE_xF_x\right);$

$u_{14} = E^2G - 2EFH - F^2G + \eta\left(EGG_x - EHF_x - FHE_x - FGF_x\right) + \eta^2\left(\frac{1}{2}GE_x^2 - HE_xF_x - \frac{1}{2}GF_x^2 + \frac{1}{2}GEE_{xx} - \frac{1}{2}FHE_{xx} - \frac{1}{2}FGF_{xx} - \frac{1}{2}EHF_{xx}\right);$

$u_{15} = 2F + \eta F_x + \frac{1}{2}\eta^2 F_{xx};$

$u_{16} = -2E - \eta E_x - \frac{1}{2}\eta^2 E_{xx};$

$u_{17} = EGH - \frac{1}{2}FG^2 - \frac{3}{2}FH^2 + 2E^2H + 2F^2H - \frac{1}{2}E^2F - \frac{1}{2}F^3 + \eta\left(\frac{1}{2}GHE_x - \frac{1}{4}G^2F_x - \frac{3}{4}H^2F_x + 2EHE_x + 2FHF_x - \frac{1}{2}EFE_x - \frac{1}{4}E^2F_x - \frac{3}{4}F^2F_x\right) + \eta^2\left(\frac{1}{4}GHE_{xx} - \frac{1}{8}G^2F_{xx} - \frac{3}{8}H^2F_{xx} + EHE_{xx} + FHF_{xx} + HE_x^2 + HF_x^2 - \frac{1}{4}EFE_{xx} - \frac{1}{8}E^2F_{xx} - \frac{3}{8}F^2F_{xx} - \frac{1}{4}FE_x^2 - \frac{3}{4}FF_x^2 - \frac{1}{2}EE_xF_x\right)$

$u_{18} = \frac{1}{2}EG^2 + \frac{3}{2}EH^2 + FGH + \frac{1}{2}E^3 + \frac{1}{2}EF^2 + \eta\left(\frac{1}{4}G^2E_x + \frac{3}{4}H^2E_x + \frac{1}{2}GHF_x + \frac{3}{4}E^2E_x + \frac{1}{4}F^2E_x + \frac{1}{2}EFF_x\right) + \eta^2\left(\frac{1}{8}G^2E_{xx} + \frac{3}{8}H^2E_{xx} + \frac{1}{4}GHF_{xx} + \frac{3}{8}E^2E_{xx} + \frac{1}{8}F^2E_{xx} + \frac{1}{4}EFF_{xx} + \frac{3}{4}EE_x^2 + \frac{1}{4}EF_x^2 + \frac{1}{2}FF_xE_x\right);$

$u_{19} = -E^2H - 2EFG + F^2H + \eta\left(-EHE_x - FGE_x - EGF_x + FHF_x\right) + \eta^2\left(-\frac{1}{2}HE_x^2 - GE_xF_x + \frac{1}{2}HF_x^2 - \frac{1}{2}HEE_{xx} - \frac{1}{2}FGE_{xx} - \frac{1}{2}EGF_{xx} + \frac{1}{2}HFF_{xx}\right).$

Appendix B

$v_0, v_1, \ldots, v_{27}$ of equations (33) - (36) are:

$v_0 = -2E - 2G + \eta G_x - \frac{1}{2}\eta^2 G_{xx};$

$v_1 = \eta\left(E + G\right) - \eta^2 G_x;$

$v_2 = -\frac{1}{2}\eta^2\left(E + G\right);$

$v_3 = \eta H - \eta^2 H_x;$

$v_4 = -\frac{1}{2}\eta^2 H;$

$v_5 = \eta\left(\frac{3}{4}E^2G + \frac{1}{4}F^2G + EG^2 + \frac{1}{4}G^3 - \frac{1}{2}EFH + EH^2 + \frac{1}{4}GH^2\right) - \eta^2\left(\frac{3}{4}E^2G_x + \frac{1}{4}F^2G_x + 2EGG_x + \frac{3}{4}G^2G_x + \frac{1}{4}H^2G_x - \frac{1}{2}EFH_x + 2EHH_x + \frac{1}{2}GHH_x\right);$

$v_6 = \eta^2\left(\frac{3}{8}E^2G + \frac{1}{8}F^2G + \frac{1}{2}EG^2 + \frac{1}{8}G^3 - \frac{1}{4}EFH + \frac{1}{2}EH^2 + \frac{1}{8}GH^2\right);$

$v_7 = \eta\left(-\frac{1}{2}EFG - \frac{3}{4}E^2H - \frac{1}{4}F^2H - \frac{1}{4}G^2H - \frac{1}{4}H^3\right) + \eta^2\left(\frac{1}{2}EFG_x + \frac{1}{2}GHG_x + \frac{3}{4}E^2H_x + \frac{1}{4}F^2H_x + \frac{1}{4}G^2H_x + \frac{3}{4}H^2H_x\right);$

$v_8 = \eta^2\left(\frac{1}{4}EFG + \frac{3}{8}E^2H + \frac{1}{8}F^2H + \frac{1}{8}G^2H + \frac{1}{8}H^3\right);$

$v_9 = E^3 + EF^2 + EG^2 - 2FGH - EH^2 - \eta\left(EGG_x - FHG_x - FGH_x - EHH_x\right) + \eta^2\left(\frac{1}{2}EG_x^2 + \frac{1}{2}EGG_{xx} - \frac{1}{2}FHG_{xx} - FG_xH_x - \frac{1}{2}EH_x^2 - \frac{1}{2}FGH_{xx} - \frac{1}{2}EHH_{xx}\right);$

$v_{10} = -\eta\left(\frac{1}{2}E^3 + \frac{1}{2}EF^2 + \frac{1}{2}EG^2 + FGH\right) + \eta^2\left(EGG_x - FHG_x - FGH_x - EHH_x\right);$

$v_{11} = \eta^2\left(\frac{1}{4}E^3 + \frac{1}{4}EF^2 + \frac{1}{4}EG^2 - \frac{1}{2}FGH - \frac{1}{4}EH^2\right);$

$v_{12} = -2F + 2H - \eta H_x + \frac{1}{2}\eta^2 H_{xx};$

$v_{13} = \eta\left(F - H\right) + \eta H_x;$

$v_{14} = \eta^2\left(-\frac{1}{2}F + \frac{1}{2}H\right);$

$v_{15} = \eta G - \eta^2 G_x;$

$v_{16} = -\frac{1}{2}\eta^2 G;$

$v_{17} = \eta^2\left(\frac{1}{2}EFG + FG^2 - \frac{1}{4}E^2 - \frac{3}{4}F^2 - \frac{1}{2}G^2 + FH^2 - \frac{1}{4}H^3\right) - \eta^2\left(EFG_x + 2FGG_x - \frac{1}{2}GHG_x - \frac{1}{4}E^2H_x - \frac{3}{4}F^2H_x - \frac{1}{4}G^2H_x + 2FHH_x - \frac{3}{4}H^2H_x\right);$

$v_{18} = \eta^2\left(\frac{1}{4}EFG + \frac{1}{2}FG^2 - \frac{1}{8}E^2H - \frac{3}{8}F^2H - \frac{1}{8}G^2H + \frac{1}{2}FH^2 - \frac{1}{8}H^3\right);$

$v_{19} = -\eta^2\left(\frac{1}{4}E^2G + \frac{3}{4}F^2G + \frac{1}{4}G^3 + \frac{1}{2}EFH + \frac{1}{4}GH^2\right) + \eta^2\left(\frac{1}{4}E^2G_x + \frac{3}{4}F^2G_x + \frac{3}{4}G^2G_x + \frac{1}{4}H^2G_x + \frac{1}{2}EFH_x + \frac{1}{2}GHH_x\right);$

$v_{20} = \eta^2\left(\frac{1}{8}E^2G + \frac{3}{8}F^2G + \frac{1}{8}G^3 + \frac{1}{4}EFH + \frac{1}{8}GH^2\right);$

$$v_{21} = E^2F + F^3 - FG^2 - 2EGH + FH^2 + \eta\left(FGG_x + EHG_x + EGH_x - FHH_x\right) + \eta^2\left(-\frac{1}{2}FG_x^2 - \frac{1}{2}FGG_{xx} - \frac{1}{2}EHG_{xx} - EG_xH_x + \frac{1}{2}FH_x^2 - \frac{1}{2}EGH_{xx} + \frac{1}{2}FHH_{xx}\right);$$

$$v_{22} = \eta\left(-\frac{1}{2}E^2F - \frac{1}{2}F^3 + \frac{1}{2}FG^2 + EGH - \frac{1}{2}FH^2\right) + \eta^2\left(-FGG_x - EHG_x - EGH_x + FHH_x\right);$$

$$v_{23} = \eta^2\left(\frac{1}{4}E^2F + \frac{1}{4}F^3 - \frac{1}{4}FG^2 - \frac{1}{2}EGH + \frac{1}{4}FH^2\right);$$

$$v_{24} = -2E - 2G - \eta E_x - \frac{1}{2}\eta^2 E_{xx};$$

$$v_{25} = E^2G - F^2G + G^3 - 2EFH + GH^2 + \eta\left(EGE_x - FHE_x - FGF_x - EHF_x\right) + \eta^2\left(-\frac{1}{2}GE_x^2 + \frac{1}{2}EGE_{xx} - \frac{1}{2}FHE_{xx} - HE_xF_x - \frac{1}{2}GF_x^2 - \frac{1}{2}FGF_{xx} - \frac{1}{2}EHF_{xx}\right);$$

$$v_{26} = 2F - 2H + \eta F_x + \frac{1}{2}\eta^2 F_{xx};$$

$$v_{27} = -2EFG - E^2H + F^2H + G^2H + H^3 + \eta\left(-FGE_x - EHE_x - EGF_x + FHF_x\right) + \eta^2\left(-\frac{1}{2}HE_x^2 - \frac{1}{2}FGE_{xx} - \frac{1}{2}EHE_{xx} - GE_xF_x + \frac{1}{2}HF_x^2 - \frac{1}{2}EGF_{xx} + \frac{1}{2}FHF_{xx}\right).$$

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