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A Mathematical Model of Iron Deficiency Anaemia and Its Impact on Cognitive Function in Young Women Aged 18-22 Years

A. Amalarathinam[1*], Sabarmathi A[2], Pooja C.[3]

[1,2,3]Department of Mathematics, Auxilium College (Autonomous), Vellore-632 006.

(Affiliated to Thiruvalluvar University, Serkkadu, Vellore-632 115, Tamil Nadu, India.)

***Corresponding Author:** Mrs. A. Amalarathinam; **E-Mail:** aamala@auxiliumcollege.edu.in

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ABSTRACT

A mathematical model of iron deficiency anaemia and its impact on cognitive function in young women aged 18-22 years is formulated. The boundedness of the system is verified using Gronwall's inequality. Equilibrium points are found. The local stability of the model is analysed around the equilibrium points. The global stability of the model is analysed using Lyapunov's theorem on stability. Numerical simulations are carried out using MATLAB.

1. Introduction

Iron is an essential mineral necessary to produce haemoglobin, the protein in red blood cells that transports oxygen throughout the body to support immune function and cognitive health [10]. The Food and Nutrition Board at the National Academies of Sciences, Engineering and Medicine has issued suggestions for iron intake that range from 8 to 27 mg for adults and 0.27 to 27 mg for younger populations. The requirements vary depending on age, sex and stage of life. However, vegetarians must consume more due to the low bioavailability of plant-based iron [11].

Iron deficiency can stem from an insufficient dietary intake of the mineral, as well as blood loss resulting from menstruation or internal haemorrhaging. When iron stores are exhausted, it leads to iron deficiency anaemia, which is characterized by fatigue, weakness and shortness of breath [9]. Iron deficiency usually progresses through three stages: storage iron depletion, iron-deficient erythropoiesis and iron deficiency anaemia [12]. Iron deficiency is the main cause of anaemia [2, 8].

Anaemia is a major worldwide public health issue, that mostly affects young children, adolescent girls and women who are experiencing periods and women who are pregnant or recently gave birth. Based on WHO estimates, 30% of women aged 15 to 49, 37% of pregnant women and 40% of infants aged 6 to 59 months are anaemic globally [13]. Numerous cognitive problems are due to deficiencies in certain micronutrients, particularly those related to iron and iodine [4]. Iron deficiency anaemia has a negative impact on the cognitive capacity of young women. In particular, there is a negative impact on activities involving arithmetic, attention span, orientation, remembering, copying, language and command tasks [1].

Many researchers have studied the impacts of iron deficiency and iron deficiency anaemia on cognitive function for various populations theoretically [3, 5, 6]. Iron deficiency anaemia has a detrimental effect on cognitive performance, particularly in young women. This study develops a mathematical model to evaluate the impact of iron deficiency anaemia on cognitive function in young women aged 18-22 years because college-aged women are more vulnerable to anaemia [7].

2. Formulation of Mathematical Model

A system of five ordinary differential equations representing the iron deficiency anaemia and its impact on cognitive function in young women aged 18-22 years is as follows:

$$\frac{dS}{dt} = \omega - \alpha S - \mu S \tag{1}$$

$$\frac{dD}{dt} = \alpha S - \mu D - \delta D - \beta D \tag{2}$$

$$\frac{dA}{dt} = \beta D - \mu A - \lambda A - \gamma A \tag{3}$$

$$\frac{dP}{dt} = \gamma A - \mu P \tag{4}$$

$$\frac{dR}{dt} = \delta D + \lambda A - \mu R \tag{5}$$

with the initial conditions $S(0) \geq 0, D(0) \geq 0, A(0) \geq 0, P(0) \geq 0, R(0) \geq 0$. Also $\omega, \alpha, \beta, \gamma, \delta, \lambda$ and μ are all positive.

Here $S(t), D(t), A(t), P(t), R(t)$ are susceptible individuals, individuals with iron deficiency, individuals with iron deficiency anaemia, individuals with poor cognitive function due to iron deficiency severe anaemia respectively. ω-Recruitment rate, α-Incidence rate of iron deficiency, β-Progression rate from iron deficiency to iron deficiency anaemia, γ-Progression rate from iron deficiency anaemia to poor cognitive function due to severe anaemia, δ-Rate of recovery from iron deficiency, λ- Rate of recovery from iron deficiency anaemia, μ −Natural death rate of women and $N(t)$-Total young women population.

The flow of the model is as follows:

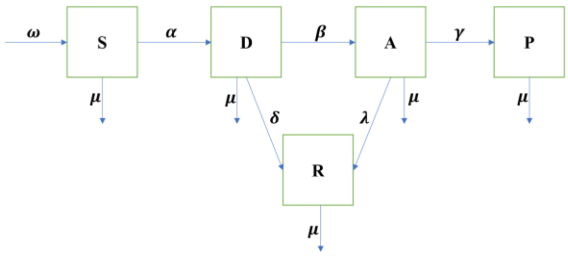

Figure 1: Mathematical Model of Iron Deficiency Anaemia and Its Impact on Cognitive Function in Young Women Aged 18-22 Years

3. Boundedness of the System

Theorem: *All the solutions of the equations (1)-(5) in R_+^5 with non-negative initial conditions are always bounded, for all $t \geq 0$.*

Proof. Let us consider $N(t) = S(t) + D(t) + A(t) + P(t) + R(t)$.

$$\frac{dN}{dt} + \theta N = \frac{dS}{dt} + \frac{dD}{dt} + \frac{dA}{dt} + \frac{dP}{dt} + \frac{dR}{dt} + \theta N$$

$\frac{dN}{dt} + \theta N \leq \omega + \theta N = \rho$(say)

By using Gronwall's inequality,

$$0 < N(t) \leq N(0)e^{-\theta t} + \frac{\rho}{\theta}(1 - e^{-\theta t})$$

When $t \to \infty$, $0 < N(t) \leq \frac{\rho}{\theta}$

Hence the solutions of the equations (1)-(5) in R_+^5 with non-negative initial conditions are always bounded, for all $t \geq 0$.

4. Equilibrium Analysis

The system has two equilibrium points. First one is the disease-free equilibrium point $E_0(\underline{S}, 0, 0, 0, 0)$ which is obtained by setting $D = A = P = R = 0$ in $\frac{dS}{dt} = 0$. That is,

$$\omega - \alpha S - \mu S = 0 \tag{6}$$

Solving (6) we get,

$$\underline{S} = \frac{\omega}{\alpha+\mu}$$

And the other is the endemic equilibrium point $E^*(S^*, D^*, A^*, P^*, R^*)$ which is obtained by setting $\frac{dS}{dt} = \frac{dD}{dt} = \frac{dA}{dt} = \frac{dP}{dt} = \frac{dR}{dt} = 0$. That is,

$$\omega - \alpha S - \mu S = 0 \tag{7}$$

$$\alpha S - \mu D - \delta D - \beta D = 0 \tag{8}$$

$$\beta D - \mu A - \lambda A - \gamma A = 0 \tag{9}$$

$$\gamma A - \mu P = 0 \tag{10}$$

$$\delta D + \lambda A - \mu R = 0 \tag{11}$$

Solving (7)-(11) we get,

$$S^* = \frac{\omega}{\alpha+\mu}, D^* = \frac{\alpha\omega}{(\alpha+\mu)(\mu+\delta+\beta)}, A^* = \frac{\alpha\beta\omega}{(\alpha+\mu)(\mu+\delta+\beta)(\mu+\lambda+\gamma)},$$

$$P^* = \frac{\gamma}{\mu}\left[\frac{\alpha\beta\omega}{(\alpha+\mu)(\mu+\delta+\beta)(\mu+\lambda+\gamma)}\right] \text{ and } R^* = \frac{1}{\mu}\left[\frac{\alpha\delta\omega(\mu+\lambda+\gamma)+\lambda\alpha\beta\omega}{(\alpha+\mu)(\mu+\delta+\beta)(\mu+\lambda+\gamma)}\right]$$

It can be noted that S^*, D^*, A^*, P^* and R^* are positive.

5. Local Stability

The Jacobian matrix of the system (1)-(5) is given by

$$J = \begin{bmatrix} -(\alpha+\mu) & 0 & 0 & 0 & 0 \\ \alpha & -(\mu+\delta+\beta) & 0 & 0 & 0 \\ 0 & \beta & -(\mu+\lambda+\gamma) & 0 & 0 \\ 0 & 0 & \gamma & -\mu & 0 \\ 0 & \delta & \lambda & 0 & -\mu \end{bmatrix}$$

At E^*, the matrix J is same as above.

The characteristic equation of J is

$$|-(\alpha+\mu)-k\ 0\ 0\ 0\ 0\ \alpha\ -(\mu+\delta+\beta)-k\ 0\ 0\ 0\ 0\ \beta\ -(\mu+\lambda+\gamma)-k\ 0\ 0\ 0\ 0\ \gamma\ -\mu-k\ 0\ 0\ \delta\ \lambda\ 0\ -\mu-k\,|=0$$

i.e. $-(k+\mu)^2(k+\alpha+\mu)(k+\beta+\delta+\mu)(k+\gamma+\mu+\lambda)=0$

The eigen values are $-\mu, -\mu, -(\alpha+\mu), -(\mu+\delta+\beta)$ and $-(\mu+\lambda+\gamma)$. Since all the eigen values are real and negative. The system is locally asymptotically stable around E^*.

6. Global Stability

$$V(S,D,A,P,R)=(S-S^*)-S^*\ln\ln\frac{S}{S^*}+(D-D^*)-D^*\ln\ln\frac{D}{D^*}+\ (A-A^*)-A^*\ln\ln\frac{A}{A^*}$$
$$+(P-P^*)-P^*\ln\ln\frac{P}{P^*}+\ (R-R^*)-R^*\ln\ln\frac{R}{R^*}$$

It can be easily shown that the function V is 0 at the equilibrium point $E^*(S^*,D^*,A^*,P^*,R^*)$ and is positive for other values of S,D,A,P and R.

The time derivative of V along the solutions of (7)-(11) is

$$\frac{dV}{dt}=\left(1-\frac{S^*}{S}\right)\frac{dS}{dt}+\left(1-\frac{D^*}{D}\right)\frac{dD}{dt}+\left(1-\frac{A^*}{A}\right)\frac{dA}{dt}+\left(1-\frac{P^*}{P}\right)\frac{dP}{dt}+\left(1-\frac{R^*}{R}\right)\frac{dR}{dt}$$

$$=\frac{dV}{dt}=-(\alpha+\mu)\frac{(S-S^*)^2}{S}-(\mu+\delta+\beta)\frac{(D-D^*)^2}{D}-(\mu+\lambda+\gamma)\frac{(A-A^*)^2}{A}-\mu\frac{(P-P^*)^2}{P}-\mu\frac{(R-R^*)^2}{R}$$

$\frac{dV}{dt}<0$

i.e. $\frac{dV}{dt}$ is negative definite.

By Lyapunov's theorem on stability, it can be concluded that the endemic equilibrium $E^*(S^*,D^*,A^*,P^*,R^*)$ is globally asymptotically stable.

7. Numerical Simulations

The parameter values for the proposed model ω,α,β and γ , were estimated using empirical data collected during a mass haemoglobin screening initiative. The programme was conducted to mark the 50th anniversary of the National Service Scheme at Auxilium College (Autonomous), Vellore, in collaboration with Naruvi Hospital, Vellore, with the objective of increasing anaemia awareness among young women. A total of 3345 young women aged 18-

22 years had their haemoglobin levels measured. Among them, 2,233 were non-anaemic, 859 had mild anaemia, 170 had moderate anaemia and 83 had severe anaemia. δ, λ and μ were assumed based on the existing literature.

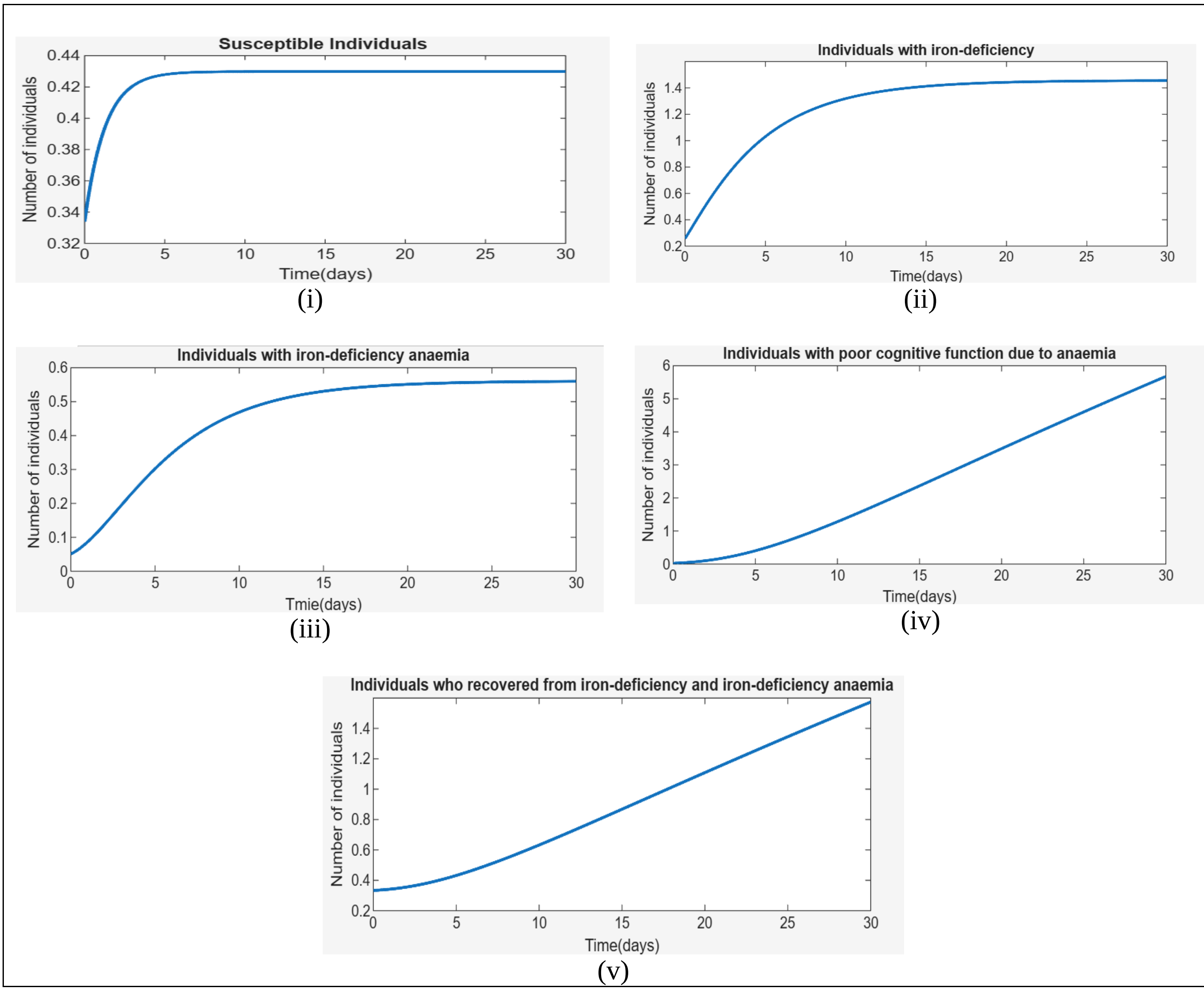

Figure 2: Flow of State Variables

In figure 2, the graphs depict the flow of state variables (i) susceptible individuals, (ii) individuals with iron deficiency, (iii) individuals with iron deficiency anaemia, (iv) individuals with poor cognitive function due to severe anaemia and (v) individuals recovered from iron deficiency and iron deficiency anaemia.

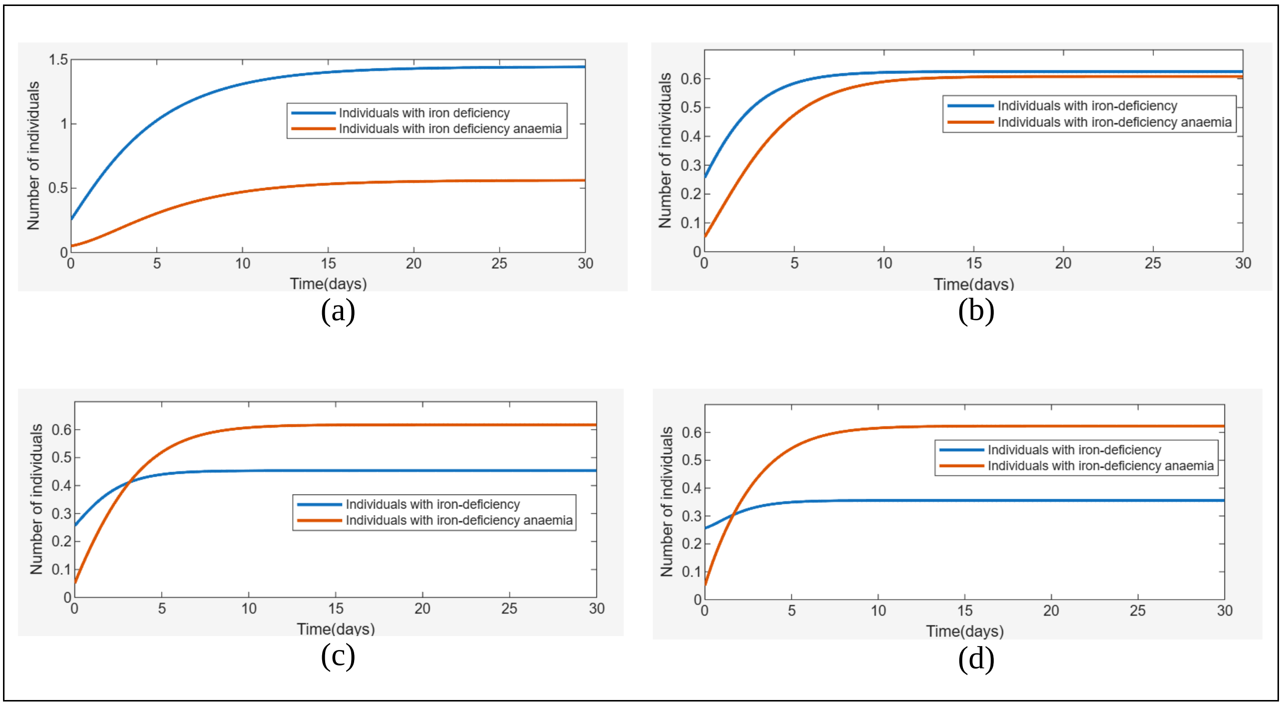

Figure 3. Flow of Individuals with Iron Deficiency and Iron Deficiency Anaemia for Different Values of β

In figure 3, the graphs display the individuals with iron deficiency and iron deficiency anaemia in four cases: (a) $\beta = 0.2$, (b) $\beta = 0.5$, (c) $\beta = 0.7$ and (d) $\beta = 0.9$. This shows that if the progression rate from iron deficiency to iron deficiency anaemia (β) increases then the number of individuals with iron deficiency anaemia increases and the number of individuals with iron deficiency decreases.

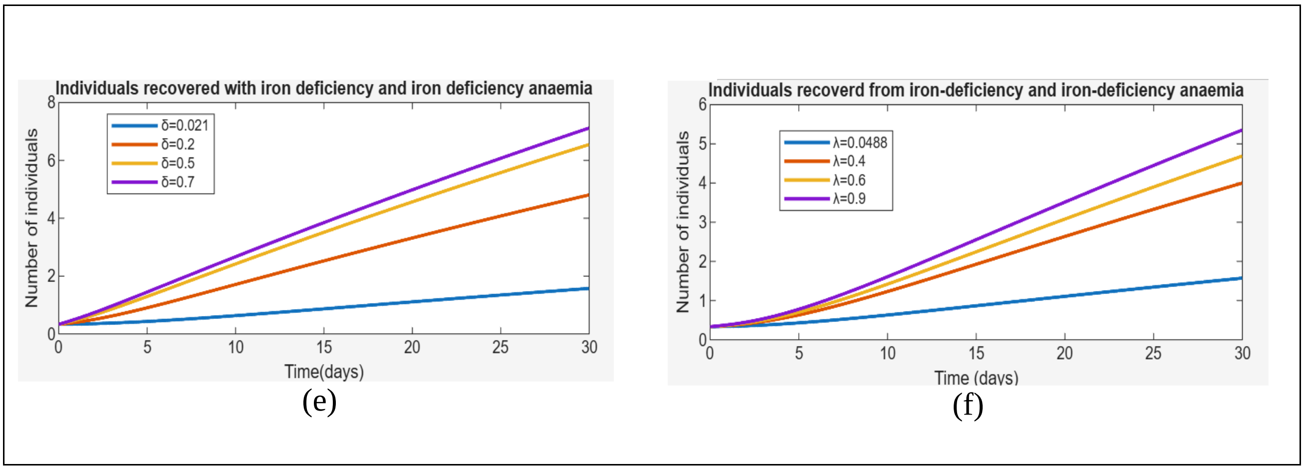

Figure 4: Individuals Recovered from Iron Deficiency and Iron Deficiency Anaemia for Different Values of δ and λ

Figure 4 shows the graphs of the individuals recovered from iron deficiency and iron deficiency anaemia. Case (e) shows that if the rate of recovery from iron deficiency(δ) increases then the number of individuals recovered from iron deficiency and iron deficiency anaemia increases. Case (f) shows that if the rate of recovery from iron deficiency anaemia (λ) increases then the number of individuals recovered from iron deficiency and iron deficiency anaemia increases.

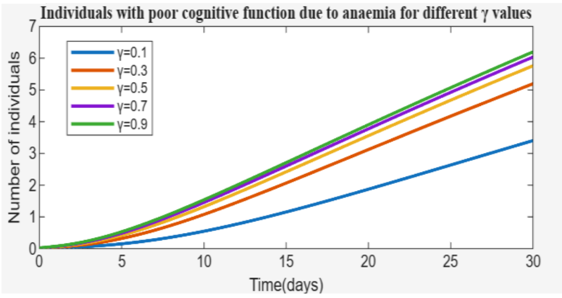

Figure 5: Individuals with Poor Cognitive Function Due to Severe Anaemia for Different Values of γ

In figure 5, the graph shows individuals with poor cognitive function due to severe anaemia for different values of γ. Whenever the progression rate from iron deficiency anaemia to poor cognitive function due to severe anaemia (γ) increases, the number of individuals with poor cognitive function due to severe anaemia increases.

8. Conclusion

This study sheds light on the dynamics of iron deficiency anaemia and its impact on cognitive function in young women between the ages 18 and 22. The model offers a strong foundation for understanding iron deficiency anaemia in this group. By focusing on limited solutions, the system's boundedness was confirmed, providing a means for effective intervention strategies. The endemic and disease-free equilibrium points were identified. The system is found to be locally asymptotically stable around the endemic equilibrium based on the eigen values and globally asymptotically stable using Lyapunov's theorem on stability. This ensures that the solutions are durable and applicable in real-world situations. Numerical simulations were carried out for the flow of all state variables, individuals with iron deficiency and iron deficiency anaemia for different values of β, individuals recovered from iron deficiency and iron deficiency anaemia for different values of δ, individuals recovered from iron deficiency and iron deficiency anaemia for different values of λ and individuals with poor cognitive function due to severe anaemia for different values of γ. Increasing β values were associated with a higher number of individuals with iron deficiency anaemia and a lower number of individuals with iron deficiency. Increasing δ values were associated with a higher number of individuals recovered from iron deficiency and iron deficiency anaemia. Similarly, increasing λ values were associated with a higher number of individuals recovered from iron deficiency and iron deficiency anaemia. Increasing γ values were associated with a higher number of individuals with poor cognitive function due to severe anaemia. These results indicate that, by reducing the progression rate from iron deficiency to iron deficiency anaemia(β), the number of individuals with iron deficiency anaemia can be reduced. As a consequence, there will be a decrease in the number of individuals with poor cognitive function due to severe anaemia. By increasing the rate of recovery from iron deficiency (δ) and the rate of recovery from iron deficiency anaemia (λ), the number of individuals recovered from iron deficiency and iron deficiency anaemia can be increased. As a consequence, there will be a decrease in the number of individuals with iron deficiency and the number of individuals with iron deficiency anaemia, which will lead to a decrease in the number of individuals with poor cognitive function due to severe anaemia. This study highlights the significance of early intervention techniques aimed at minimizing the progression of iron deficiency anaemia and enhancing recovery rates through rapid detection and efficient medical care in young women. All things considered, the model offers insightful information on managing iron deficiency anaemia and its impact on cognitive function in young women. It may also be a helpful framework for guiding future research and understanding the importance of public health initiatives aimed at young women.

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