

## Tribonacci L-cordial labeling of some standard graphs

P. Hameetha Begum<sup>1, a</sup> M. Amirthakodi<sup>2, b</sup> S. Akila Devi<sup>3, c</sup>

<sup>1</sup>Research Scholar, Reg.No: 22222102092011, PG and Research Department of Mathematics, Kamaraj College Thoothukudi (Affiliated to Manonmaniam Sundaranar University, Tirunelveli), Tamil Nadu, India & Assistant Professor, Department of Mathematics, A.D.M College For Women, Nagapattinam, Tamil Nadu, India

<sup>2</sup> Department of Mathematics, Kamaraj College, Thoothukudi, Tamil Nadu, India

<sup>3</sup> Assistant Professor, Department of Mathematics, P.S.R Engineering College, Sivakasi, Tamil Nadu, India

<sup>a)</sup>[p.hameethabegum@adjadmc.ac.in](mailto:p.hameethabegum@adjadmc.ac.in)

<sup>b)</sup>[amirthakodim@yahoo.com](mailto:amirthakodim@yahoo.com)

<sup>c)</sup>[apakilamaths@gmail.com](mailto:apakilamaths@gmail.com)

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### Abstract:

In this article, we introduce Tribonacci L-cordial labeling and find the existence of the same on some standard graphs.

## 1. Introduction:

Graph Labeling [7] was first introduced by Alexander Rosa in 1967 that strongly relates Algebra and structure of graphs. It is generally used in communication networks. The concept of cordial labeling [1] & [9] of graphs was introduced by Cahit in 1987. In this whole work,  $G$  is considered as a  $(V(G), E(G))$  graph that is simple, finite, connected and undirected graph with  $p$  vertices and  $q$  edges [3] & [6].

Meena and Nagarajan introduced the concept of intersection cordial labeling (ICL) [4] & [8] proved its existence on standard graphs.

Inspired from the ICL, we proved the existence of the same in some regular graphs[6] and defined a new labeling called union cordial labeling(UCL) [5] and also proved its existence in standard graphs and tree related graphs. Moreover we again define a new labeling namely Tribonacci L-cordial labeling (TL-cordial Labeling) based on least common multiple (LCM) concept and proved its existence on some standard graphs such as paths, cycles, trees, combs, twigs, H-graphs, coconut trees, Y-trees[2], stars and star related graphs.

### Definition: 1.1

Let  $G$  be a  $(p,q)$  graph. The tribonacci sequence is a generalization of the Fibonacci sequence where each term is the sum of the three preceeding terms. The tribonacci sequence was first described by Agronomof in 1914 but it was first used in the origin of Species by Charles R.

The first few terms of the tribonacci sequence are 0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149,... The numbers are represented as  $T_0, T_1, T_2, \dots$ . On the whole, we neglect the initial values  $T_0 = 0$  and  $T_1 = 1$ . Therefore, we take the sequence as  $T_1 = 1, T_2 = 2, T_3 = 4, \dots$

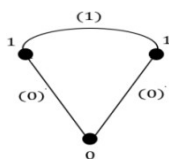
Let  $T$  denotes the set of tribonacci numbers. Let  $f: V(G) \rightarrow \{T_1, T_2, \dots, T_p\}$  be a bijection. Then the induced edge function  $f^*: E(G) \rightarrow \{0,1\}$  defined by

$$f^*(uv) = \begin{cases} 1 & \text{if LCM}(f(u), f(v)) \text{ is odd} \\ 0 & \text{if LCM}(f(u), f(v)) \text{ is even} \end{cases}$$

is called **Tribonacci L-cordial labeling** (TL-cordial Labeling) if  $|e_f(0) - e_f(1)|$  is either 0 or 1. A graph which admits TL-cordial labeling is said to be a **TL-cordial graph**.

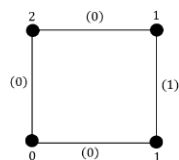
**Note:** Here  $e_f(0)$  stands for the number of edges labeled with 0 and  $e_f(1)$  stands for the number of edges labeled with 1.

**Example: 1.2** Consider the following graph:



From the above vertex labeling,  $e_f(0) = 2$  and  $e_f(1) = 1 \Rightarrow |e_f(0) - e_f(1)| = 2 - 1 = 1$ . Hence the graph admits TL-cordial Labeling.

**Example: 1.3** Consider the graph  $C_4$  :



From the above vertex labeling,  $e_f(0) = 3$  and  $e_f(1) = 1$ .

Hence  $|e_f(0) - e_f(1)| = 3 - 1 = 2$  and therefore,  $C_4$  is not a TLC graph.

## 2. Main Theorems:

### Theorem: 2.1

Cycles  $C_n$ , admit TL-cordial Labeling for  $n = 4k + 1$ ,  $k = 1, 2, 3, \dots$

#### Proof:

Let  $V(G) = \{u_1, u_2, \dots, u_n\}$  be the vertices of  $C_n$ .

Label the vertices  $u_1, u_2, \dots, u_{\frac{n+1}{2}}$  of  $C_n$  by the odd tribonacci numbers and label  $u_{\frac{n+1}{2}+1}, \dots, u_n$  by the even tribonacci numbers in order. Then we get  $e_f(0) = \frac{n+1}{2}$  and  $e_f(1) = \frac{n+1}{2} - 1$  and

hence  $|e_f(0) - e_f(1)| = \frac{n+1}{2} - (\frac{n+1}{2} - 1)$  which results as  $|e_f(0) - e_f(1)| = 1$ .

Therefore, cycles admit TL-cordial Labeling for  $n = 4k + 1$ ,  $k = 1, 2, 3, \dots$

### Theorem: 2.2

Paths  $P_n$ ,  $n \geq 3$  admit TL-cordial Labeling for  $n \neq 4k - 1$ ,  $k = 1, 2, 3, \dots$

#### Proof:

Let  $V(G) = \{u_1, u_2, \dots, u_n\}$  be the vertices of  $P_n$ .

**Case (i):**  $n = 2k + 2$  for  $k = 1, 2, 3, \dots$

Let  $u_1, u_2, \dots, u_{2k+2}$  be the vertices of the path. Label  $u_1, u_{2k+2}, u_2, u_{2k+1}, u_3, u_{2k}, \dots$  by the even  $T_n$ 's in order. Label the remaining vertices by the odd  $T_n$ 's so that the edge labeling results as the number of zero's one greater than number of one's. Hence  $|e_f(0) - e_f(1)| = 1$ .

Therefore, Path admits TL-cordial Labeling for  $n = 2k + 2$  for  $k = 1, 2, 3, \dots$

**Case (ii):**  $n = 4k + 1$  for  $k = 1, 2, 3, \dots$

As  $n = 4k + 1$ , there are odd number of vertices in  $P_n$ .

Label the vertices as in case (i) (i.e.,) Label the even  $T_n$ 's for the end vertices and their neighbours in order and the odd  $T_n$ 's for the remaining intermediate vertices so that the edge labeling results as the number of zero's equal to number of one's.

Hence  $|e_f(0) - e_f(1)| = 0$ .

Therefore, Path admits TL-cordial Labeling for  $n = 4k + 1$  for  $k = 1, 2, 3, \dots$

### Corollary: 2.3

Path  $P_n$  does not admit TL-cordial Labeling for  $n = 4k - 1$ ,  $k = 1, 2, 3, \dots$

The result does not exist because in case of  $n = 4k - 1$ , the cardinality of even  $T_n$ 's is greater than the cardinality of odd  $T_n$ 's resulting in an absolute difference greater than 1.

### Corollary: 2.4

- (i) Let  $G \cong P_n$ ,  $n = 4k + 1, k = 1, 2, 3, \dots$ . The addition of a pendant edge to either an internal vertex or a pendant vertex of  $G$  admits TL-cordial Labeling.
- (ii) Let  $G \cong P_n$ ,  $n = 4k + 2, k = 1, 2, 3, \dots$ . The addition of a pendant edge to either an internal vertex or a pendant vertex of  $G$  does not admit TL-cordial Labeling.

In the case of  $n = 4k + 2$ , the cardinality of even  $T_n$ 's is greater than that of odd  $T_n$ 's resulting in an absolute difference greater than 1 and hence  $G$  does not admit TL-cordial Labeling.

- (iii) Let  $G \cong P_n$ ,  $n = 4k, k = 1, 2, 3, \dots$ . Addition of a pendant edge to the internal vertex of  $G$  admits TL-cordial Labeling when it is labeled with immediate tribonacci number. Otherwise, it follows as in the case of **Corollary: 2.4 (ii)**

### Theorem: 2.5

Comb graphs  $P_n \odot K_1$ ,  $n \geq 3$  are TLC graphs.

#### Proof:

Let  $u_1, u_2, \dots, u_n$  be the vertices of the path  $P_n$  and  $v_1, v_2, \dots, v_n$  be the pendant vertices adjacent to the vertices in the path respectively.

Label the path vertices with the odd tribonacci numbers and label the pendant vertices with the even tribonacci numbers.

Then we get the edge labeling as  $e_f(1) = n - 1$  and  $e_f(0) = n$  so that

$$|e_f(0) - e_f(1)| = |n - (n - 1)| = 1.$$

Therefore, comb graphs are TLC graphs.

### Corollary: 2.6

- (i) Let  $G \cong P_n \odot K_1$ ,  $n = 2k, k = 2, 3, \dots$ . The addition of a pendant edge to a path vertex, that is labeled with odd tribonacci number, of  $G$  admits TL-cordial Labeling.
- (ii) Let  $G \cong P_n \odot K_1$ ,  $n = 2k + 1, k = 1, 2, 3, \dots$ . The addition of a pendant edge to any vertex of  $G$  does not admit TL-cordial Labeling.

In the case of  $n = 2k + 1, k = 1, 2, 3, \dots$ , the cardinality of even  $T_n$ 's is greater than that of odd  $T_n$ 's resulting in the absolute difference greater than 1 and hence  $G$  does not admit TL-cordial Labeling.

### Theorem: 2.7

Star graphs  $K_{1,q}$ , are TLC graphs except for  $q = 4n + 2, n = 1, 2, 3, \dots$

#### Proof:

Let  $v_1, v_2, \dots, v_p$  be the vertices of star graph and let it has  $q$  edges.

Let  $v_p$  be the central vertex and  $v_1, v_2, \dots, v_{p-1}$  be the pendant vertices of the star  $K_{1,q}$ .

Label the central vertex  $v_p$  by  $T_1$ (i.e.,) 1 and label the pendant vertices by the succeeding  $T_n$ 's.

**Case (i):**  $q = 2n + 1$  for  $n = 1, 2, 3, \dots$

As  $q = 2n + 1$ , there are even number of vertices in  $K_{1,q}$ .

Label the central vertex  $v_p$  by  $T_1$ (i.e.,) 1 and label the pendant vertices by the succeeding  $T_n$ 's so that the absolute difference of the number of zero's and the number of one's, differ by 1. Hence  $|e_f(0) - e_f(1)| = 1$ .

Therefore, star graphs admit TL-cordial Labeling for  $q = 2n + 1$  for  $n = 1, 2, 3, \dots$

**Case (ii):**  $q = 4n$  for  $n = 1, 2, 3, \dots$

As  $q = 4n$ , there are odd number of vertices in  $K_{1,q}$ .

Label the vertices as in the previous case so that the edge labeling results as the number of zero's equal to number of one's. Hence  $|e_f(0) - e_f(1)| = 0$ .

Therefore, star graphs admit TL-cordial Labeling for  $q = 4n$  for  $n = 1, 2, 3, \dots$

### Corollary: 2.8

Star graphs  $K_{1,q}$  do not admit TL-cordial Labeling for  $q = 4n + 2$ ,  $n = 1, 2, 3, \dots$

The result does not exist because in case of  $q = 4n + 2$ , the cardinality of even  $T_n$ 's is greater than the cardinality of odd  $T_n$ 's resulting in an absolute difference greater than 1.

### Corollary: 2.9

- (i) Let  $G \cong K_{1,q}$ ,  $q = 4n - 1$ ,  $n = 1, 2, 3, \dots$ . The addition of a pendant edge to any vertex of  $G$  admits TL-cordial Labeling.
- (ii) Let  $G \cong K_{1,q}$ ,  $q = 4n$ ,  $n = 1, 2, 3, \dots$ . The addition of a pendant edge to any vertex of  $G$  admits TL-cordial Labeling.
- (iii) Let  $G \cong K_{1,q}$ ,  $q = 4n + 1$ ,  $n = 1, 2, 3, \dots$ . The addition of a pendant edge to any vertex of  $G$  does not admit TL-cordial Labeling.

In the case of  $q = 4n + 1$ , the cardinality of even  $T_n$ 's is greater than that of odd  $T_n$ 's resulting in an absolute difference greater than 1 and hence  $G$  does not admit TL-cordial Labeling.

### Theorem: 2.10

Bistar graphs,  $B_{m,m}$ ,  $m \geq 2$  admit TL-cordial Labeling.

#### Proof:

Let the bistar graph  $B_{m,m}$  be a  $(2m + 2, 2m + 1)$  graph.

Let  $xy$  be the edge joining the two stars  $K_{1,m}$  and  $K_{1,m}$ .

Let  $u_1, u_2, \dots, u_m$  be the vertices adjacent to  $x$  and  $u_{m+1}, u_{m+2}, \dots, u_{2m}$  be the vertices adjacent to  $y$ .

Label  $x$  and  $y$  by  $T_1$  and  $T_2$  which contributes one '0' to  $e_f(0)$ . Label the vertices  $u_1, u_2, \dots, u_m$  by the odd  $T_n$ 's and  $u_{m+1}, u_{m+2}, \dots, u_{2m}$ , by the even  $T_n$ 's so that it contributes 'm' 0's and 'm' 1's which implies  $e_f(0) = m + 1$  and  $e_f(1) = m$ .

Hence  $|e_f(0) - e_f(1)| = 1$  providing  $B_{m,m}$ , a TLC graph.

### Corollary: 2.11

- (i) Let  $G \cong B_{m,m}$ ,  $m = 2k + 1$ ,  $k = 1, 2, 3, \dots$ . The addition of a pendant edge to vertex 'x' of  $G$  admits TL-cordial Labeling.
- (ii) Let  $G \cong B_{m,m}$ ,  $m = 2k$ ,  $k = 1, 2, 3, \dots$ . The addition of a pendant edge to any vertex of  $G$  does not admit TL-cordial Labeling.

The result does not exist because in case of  $m = 2k$  in  $G$ , the cardinality of even  $T_n$ 's is greater than the cardinality of odd  $T_n$ 's resulting in an absolute difference greater than 1.

**Theorem: 2.12**

Y- trees  $Y_n$  [1] are TLC graphs except for  $n = 4k + 1$ ,  $k = 1, 2, 3, \dots$

**Proof:**

Let Y- tree,  $Y_n$  be a  $(n + 2, n + 1)$  graph with  $V(G) = \{v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}\}$  and  $E(G) = \{v_i v_{i+1}, i = 1, 2, 3, \dots, (n - 1)\} \cup \{v_n v_{n+1}\} \cup \{v_n v_{n+2}\}$ .

Define  $f: V(G) \rightarrow \{T_1, T_2, \dots, T_{n+2}\}$

**Case(i):**  $n = 2k$  for  $k = 2, 3, \dots$

Assign even  $T_n$ 's to first  $k + 1$  vertices of  $P_n$  and odd  $T_n$ 's to the remaining vertices contributing ' $k + 1$ ' 0's and ' $k$ ' 1's that yields  $|e_f(0) - e_f(1)| = 1$ .

Hence  $Y_n$  are TLC graphs for  $n = 2k$  for  $k = 2, 3, \dots$

**Case(ii):**  $n = 4k - 1$  for  $k = 1, 2, 3, \dots$

Assign even  $T_n$ 's to first  $2k$  vertices of  $P_n$  and odd  $T_n$ 's to the remaining  $2k + 1$  vertices contributing ' $2k$ ' 0's and ' $2k + 1$ ' 1's that yields  $|e_f(0) - e_f(1)| = 0$ .

Hence  $Y_n$  are TLC graphs for  $n = 4k - 1$  for  $k = 1, 2, 3, \dots$

**Corollary: 2.13**

Y- trees  $Y_n$  do not admit TL-cordial Labeling for  $n = 4k + 1$ ,  $k = 1, 2, 3, \dots$

The result does not exist because in case of  $n = 4k + 1$ , the cardinality of even  $T_n$ 's is greater than the cardinality of odd  $T_n$ 's resulting in an absolute difference greater than 1.

**Corollary: 2.14**

- (i) Let  $G \cong Y_n$ ,  $n = 4k + 2$ ,  $k = 1, 2, 3, \dots$ . The addition of a pendant edge to a pendant vertex, which is labeled with odd tribonacci number, of  $G$  admits TL-cordial Labeling.
- (ii) Let  $G \cong Y_n$ ,  $n = 4k - 1$ ,  $k = 1, 2, 3, \dots$ . The addition of a pendant edge to any vertex of  $G$  admits TL-cordial Labeling.
- (iii) Let  $G \cong Y_n$ ,  $n = 4k$  &  $4k + 1$ ,  $k = 1, 2, 3, \dots$ . Addition of a pendant edge to any vertex of  $G$  does not admit TL-cordial Labeling.

In the case of  $n = 4k$  &  $4k + 1$ , the cardinality of even  $T_n$ 's is greater than that of odd  $T_n$ 's resulting in an absolute difference greater than 1 and hence  $G$  does not admit TL-cordial Labeling.

**Theorem: 2.15**

Twig graphs  $T_n$ ,  $n \geq 3$  admit TL-cordial Labeling for  $n \neq 4k + 1$ ,  $k = 1, 2, 3, \dots$

**Proof:**

Let the Twig graph  $T_n$  be a  $(3n - 4, 3n - 5)$  graph.

Let  $u_1, u_2, \dots, u_n$  be the vertices of the path;  $u_1', u_2', \dots, u_{n-2}'$  be the pendant vertices attached above the internal vertices of the path and  $u_1'', u_2'', \dots, u_{n-2}''$  be the pendant

vertices attached below the internal vertices of the path.

Define  $f: V(G) \rightarrow \{T_1, T_2, \dots, T_{3n-4}\}$ .

If  $n = 3$ ,  $T_3$  is isomorphic to  $K_{1,4}$  which is a TLC graph. (Refer case(ii) of Theorem:2.4).

**Case(i):**  $n = 2k + 2$ ,  $k = 1, 2, 3, \dots$

- ❖ Label the vertices  $u_1, u_2, \dots, u_n$  by the odd  $T_n$ 's in the successive order so that it contributes  $(n - 1)1$ 's to  $e_f(1)$ .
- ❖ Label the vertices  $u_1', u_2', \dots, u_{n-2}'$  by the first  $(n - 2)$  even  $T_n$ 's so that it contributes  $(n - 2)0$ 's to  $e_f(0)$ .

Label the vertices  $u_1'', u_2'', \dots, u_{n-2}''$  by the remaining  $T_n$ 's in the order so that it contributes  $\frac{n}{2}0$ 's and  $\frac{n-4}{2}1$ 's. Now,  $e_f(0) = n - 2 + \frac{n}{2} = \frac{3n-4}{2}$  and  $e_f(1) = n - 1 + \frac{n-4}{2} = \frac{3n-6}{2}$  and we get  $|e_f(0) - e_f(1)| = \frac{3n-4}{2} - \frac{3n-6}{2} = 1$ .

Therefore, Twig graphs admit TL-cordial Labeling for  $n = 2k + 2$ ,  $k = 1, 2, 3, \dots$ .

**Case(ii):**  $n = 4k + 3$ ,  $k = 1, 2, 3, \dots$

- ❖ Label the vertices  $u_1, u_2, \dots, u_n$  by the odd  $T_n$ 's in the successive order so that it contributes  $(n - 1)$  one's to  $e_f(1)$ .
- ❖ Label the vertices  $u_1', u_2', \dots, u_{n-2}'$  by the first  $n - 2$  even  $T_n$ 's so that it contributes  $(n - 2)$  zero's to  $e_f(0)$ .

Label the vertices  $u_1'', u_2'', \dots, u_{n-2}''$  by the remaining  $T_n$ 's in the successive order so that it contributes  $\frac{n}{2}$  zero's and  $\frac{n-2}{2}$  one's. Now,  $e_f(0) = n - 2 + \frac{n}{2} = \frac{3n-4}{2}$  and  $e_f(1) = n - 1 + \frac{n-2}{2} = \frac{3n-4}{2}$  and we get  $|e_f(0) - e_f(1)| = \frac{3n-4}{2} - \frac{3n-4}{2} = 0$ .

Therefore, Twig graphs admit TL-cordial Labeling for  $n = 4k + 3$ ,  $k = 1, 2, 3, \dots$

### Corollary: 2.16

Twig graphs  $T_n$ ,  $n \geq 3$  do not admit TL-cordial Labeling for  $n = 4k + 1$ ,  $k = 1, 2, 3, \dots$

The result does not exist because in case of  $n = 4k + 1$ , the cardinality of even  $T_n$ 's is greater than the cardinality of odd  $T_n$ 's resulting in an absolute difference greater than 1.

### Corollary: 2.17

- (i) Let  $G \cong T_n$ ,  $n \geq 3$ ,  $n = 4k$ ,  $k = 1, 2, 3, \dots$  The addition of a pendant edge to any vertex labeled with odd  $T_n$ 's of  $G$  admits TL-cordial Labeling.
- (ii) Let  $G \cong T_n$ ,  $n \geq 3$ ,  $n = 4k + 3$ ,  $k = 1, 2, 3, \dots$  The addition of a pendant edge to any vertex of  $G$  admits TL-cordial Labeling.
- (iii) Let  $G \cong T_n$ ,  $n \geq 3$ ,  $n = 4k + 2$ ,  $k = 1, 2, 3, \dots$  The addition of a pendant edge to any vertex of  $G$  does not admit TL-cordial Labeling.

In the case of  $n = 4k + 2$ , the cardinality of even  $T_n$ 's is greater than that of odd  $T_n$ 's resulting in an absolute difference greater than 1 and hence  $G$  does not admit TL-cordial Labeling.

### Definition: 2.18

The H- graph of a path  $P_n$ , denoted by  $H_n$  is the graph obtained from two copies of  $P_n$  with vertices  $u_1, u_2, \dots, u_n$  and  $v_1, v_2, \dots, v_n$  by joining the vertices  $u_{\frac{(n+1)}{2}}$  and  $v_{\frac{(n+1)}{2}}$ , if  $n$  is odd and the vertices  $u_{\frac{(n)}{2}}$  and  $v_{\frac{(n)}{2}+1}$ , if  $n$  is even.

### Theorem: 2.19

H-graphs  $H_k$ ,  $k \geq 3$  are TLC graphs.

#### Proof:

Let the H-graph  $H_k$  be a  $(2n, 2n - 1)$  graph.

Let  $V(G) = u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$  and

$E(G) = \{u_1u_2, u_2u_3, \dots, u_{n-1}u_n\} \cup \{v_1v_2, v_2v_3, \dots, v_{n-1}v_n\} \cup \{u_{\frac{(n+1)}{2}}v_{\frac{(n+1)}{2}}\}$  if  $n$  is odd  
and

$E(G) = \{u_1u_2, u_2u_3, \dots, u_{n-1}u_n\} \cup \{v_1v_2, v_2v_3, \dots, v_{n-1}v_n\} \cup \{u_{\frac{(n)}{2}}v_{\frac{(n)}{2}+1}\}$  if  $n$  is even.

Label the vertices  $u_1, u_2, \dots, u_n$  by the even  $T_n$ 's in the order so that it contributes  $(n - 1)$  zero's to  $e_f(0)$  and label the vertices  $v_1, v_2, \dots, v_n$  by the odd  $T_n$ 's in the order so that it contributes  $(n - 1)$  one's to  $e_f(1)$ .

As  $u_i$ 's,  $i = 1, 2, 3, \dots, n$  are labeled with even  $T_n$ 's and  $v_i$ 's,  $i = 1, 2, 3, \dots, n$  are

labeled with odd  $T_n$ 's, both the edges  $u_{\frac{(n+1)}{2}}v_{\frac{(n+1)}{2}}$  or  $u_{\frac{(n)}{2}}v_{\frac{(n)}{2}+1}$  induces one zero to  $e_f(0)$  in both the cases (i.e.,) either the value of  $n$  is odd or even. Now  $e_f(0) = n - 1 + 1 = n$  and  $e_f(1) = n - 1$  and we get  $|e_f(0) - e_f(1)| = n - (n - 1) = 1$

Therefore,  $H_k$ ,  $k \geq 3$  are TLC graphs.

### Corollary: 2.20

Let  $G \cong H_k$ ,  $k \geq 3$ . The addition of a pendant edge to any vertex labeled with odd  $T_n$ 's of  $G$  admits TL-cordial Labeling.

### Definition: 2.21

A coconut tree  $CT(m, n)$  is the graph obtained from the path  $P_m$  by appending  $n$  new pendant edges at an end vertex of  $P_m$ .

### Theorem: 2.22

Coconut tree  $CT(m, n)$ , are TLC graphs except for  $m + n = 4k + 3, k = 1, 2, 3, \dots$

#### Proof:

Let the Coconut tree  $CT(m, n)$  be a  $(m + n, m + n - 1)$  graph.

Let  $u_1, u_2, \dots, u_m$  be the vertices of the path  $P_m$  and  $v_1, v_2, \dots, v_n$  be the vertices appended to the end vertex  $u_m$  of the path  $P_m$  and the edges are  $u_2u_3, \dots, u_{n-1}u_m, u_mv_1, u_mv_2, \dots, u_mv_n$ .

Label the end vertex  $u_m$  of the path by  $T_1$  (i.e.,) 1 and the pendant vertices  $v_1, v_2, \dots, v_n$  by the odd  $T_n$ 's succeeding 1. If all the pendant vertices got labeled with the odd  $T_n$ 's, then label the remaining path vertices by the even  $T_n$ 's. If suppose, all the pendant vertices do not get labeled with the odd  $T_n$ 's, then label all the remaining vertices (i.e.,) both unlabeled pendant vertices and



path vertices by the even  $T_n$ 's.

Now, we get either  $|e_f(0) - e_f(1)| = 0$  or  $|e_f(0) - e_f(1)| = 1$ .

Therefore,  $CT(m, n)$  are TLC graphs except for  $m + n = 4k + 3, k = 1, 2, 3, \dots$

### Corollary: 2.23

$CT(m, n)$  do not admit TL-cordial Labeling for  $m + n = 4k + 3, k = 1, 2, 3, \dots$

In the case of  $m + n = 4k + 3$ , the cardinality of even  $T_n$ 's is greater than that of odd  $T_n$ 's resulting in an absolute difference greater than 1 and hence  $G$  does not admit TLC labeling.

### Corollary: 2.24

Let  $G \cong CT(m, n)$ , The addition of a pendant edge to any vertex of  $G$  labeled with odd  $T_n$ 's admits TL-cordial Labeling.

### Conclusion:

In this work, we proved the existence of TL-cordial Labeling of some standard graphs. Proceeding this, we will work to prove the same for some special classes of graphs.

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