

Tribonacci L-cordial labeling of some standard graphs

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Abstract:

In this article, we defined Tribonacci L-cordial labeling and find the existence of the same on some standard graphs.

I. Introduction:

Graph Labeling [7] was first introduced by Alexander Rosa in 1967 that strongly relates Algebra and structure of graphs. It is generally used in communication networks. The concept of cordial labeling [1] & [9] of graphs was introduced by Cahit in 1987. In this whole work, G is considered as a $(V(G), E(G))$

graph that is simple, finite, connected and undirected graph with p vertices and q edges [3] & [6].

Meena and Nagarajan introduced the concept of intersection cordial labeling (ICL) [4] & [8] proved its existence on standard graphs. Inspired from the ICL, we proved the existence of the same in some regular

graphs[6] and defined a new labeling called union cordial labeling(UCL) [5] and also proved its existence in standard graphs and tree related graphs. Moreover we again define a new labeling namely Tribonacci L-cordial labeling (**TL-cordial Labeling**)

based on least common multiple (LCM) concept and proved its existence on some standard graphs such as paths, cycles, trees, combs, twigs, H-graphs, coconut trees, Y-trees[2], stars and star related graphs.

Definition: 1.1

Let G be a (p,q) graph. The tribonacci sequence is a generalization of the Fibonacci sequence where each term is the sum of the three preceding terms. The tribonacci sequence was first described by Agronomof in 1914 but it was first used in the origin of Species by Charles R.

The first few terms of the tribonacci sequence are 0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149,... The numbers are represented as $t_0, t_1, t_2, \dots$. On the whole, we neglect the initial values $t_0 = 0$ and $t_1 = 1$. Therefore, we take the sequence as $t_1 = 1, t_2 = 2, t_3 = 4, \dots$

Let T denotes the set of tribonacci numbers. Let $f: V \rightarrow \{t_1, t_2, \dots\}$ be a bijection. Then the induced edge function $^*(f): E \rightarrow \{0,1\}$ defined by

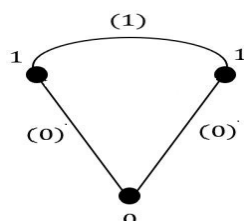
$$^*(uv) = \begin{cases} 1 & \text{if LCM}(f(u), f(v)) \text{ is odd} \\ 0 & \text{if LCM}(f(u), f(v)) \text{ is even} \end{cases}$$

is called Tribonacci L-cordial labeling(TL-cordial Labeling) if $| \sum_{e \in E} ^*(e) - \sum_{e \in E} ^*(e) |$ is either 0 or 1. A graph which admits TL-cordial labeling is said to be a TL-Cordial graph.

Note: Here (0) stands for the number of edges labeled with 0 and (1) stands for the number of edges labeled with 1.

Example: 1.2

Consider the following graph:

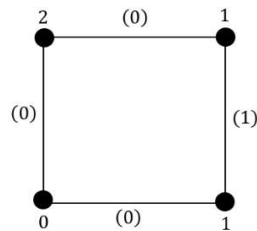


From the above vertex labeling, $(0) = 2$ and $(1) = 1 \Rightarrow | \sum_{e \in E} ^*(e) - \sum_{e \in E} ^*(e) | = 2 - 1 = 1$.

Hence the graph admits TL-cordial Labeling.

Example: 1.3

Consider the graph C_4 :



From the above vertex labeling, $(0) = 3$ and $(1) = 1$.

Hence $|(0) - (1)| = 3 - 1 = 2$ and therefore, C_4 is not a TLC graph.

2. Main Theorems:

Theorem: 2.1

Cycles C_n , admit TL-cordial Labeling for $n = 4k + 1$, $k = 1, 2, 3, \dots$

Proof:

Let $V(G) = \{u_1, u_2, \dots, u_n\}$ be the vertices of.

Label the vertices $1, 2, \dots, \pm 1$ of C_n by the odd tribonacci numbers

and label $\pm 1, \dots$ by the even tribonacci numbers in order.

Then we get $(0) = \frac{+1}{2}$ and $(1) = \frac{+1 - 1}{2}$

hence $|(0) - (1)| = \frac{+1}{2} - \left(\frac{+1}{2} - 1\right)$ which results as $|(0) - (1)| = 1$.

Therefore, cycles admit TL-cordial Labeling for $n = 4k + 1$, $k = 1, 2, 3, \dots$

Theorem: 2.2

Paths P_n , $n \geq 3$ admit TL-cordial Labeling for $n \neq 4k - 1$, $k = 1, 2, 3, \dots$

Proof:

Let $V = \{1, 2, \dots\}$ be the vertices of.

Case (i): $n = 2k + 2$ for $k = 1, 2, 3, \dots$

Let $u_1, u_2, \dots, u_{2k+2}$ be the vertices of the path. Label $u_1, u_{2k+2}, u_2, u_{2k+1}, u_3, u_{2k}, \dots$ by the

even T_n 's in order. Label the remaining vertices by the odd T_n 's so that the edge labeling

results as the number of zero's one greater than number of one's.

Hence $|e_f(0) - e_f(1)| = 1$.

Therefore, Path admits TL-cordial Labeling for $n = 2k + 2$ for $k = 1, 2, 3, \dots$.

Case (ii): $n = 4 + 1$ for $n = 1, 2, 3, \dots$

As $n = 4 + 1$, there are odd number of vertices in.

Label the vertices as in case (i) (i.e.,) Label the even T_n 's for the end vertices and their neighbours in order and the odd T_n 's for the remaining intermediate vertices so that the edge labeling results as the number of zero's equal to number of one's.

Hence $|(0) - (1)| = 0$.

Therefore, Path admits TL-cordial Labeling for $n = 4 + 1$ for $k = 1, 2, 3, \dots$

Corollary: 2.3

Path P_n does not admit TL-cordial Labeling for $n = 4k - 1$, $k = 1, 2, 3, \dots$

The result does not exist because in case of $n = 4k - 1$, the cardinality of even T_n 's is greater than the cardinality of odd T_n 's resulting in an absolute difference greater than 1.

Corollary: 2.4

- (i) Let $\cong P_n$, $n = 4k + 1$, $= 1, 2, 3, \dots$. The addition of a pendant edge to either an internal vertex or a pendant vertex of G admits TL-cordial Labeling.
- (ii) Let $\cong P_n$, $n = 4k + 2$, $= 1, 2, 3, \dots$. The addition of a pendant edge to either an internal vertex or a pendant vertex of G does not admit TL-cordial Labeling.

In the case of $= 4 + 2$, the cardinality of even T_n 's is greater than that of odd T_n 's resulting in an absolute difference greater than 1 and hence G does not admit TL-cordial Labeling.

- (iii) Let $\cong P_n$, $n = 4k$, $= 1, 2, 3, \dots$. Addition of a pendant edge to the internal vertex of G admits TL-cordial Labeling when it is labeled with immediate tribonacci number. Otherwise, it follows as in the case of **Corollary: 2.4 (ii)**

Theorem: 2.5

Comb graphs $P_n \odot K_1$, $n \geq 3$ are TLC graphs.

Proof:

Let $1, 2, \dots$, be the vertices of the path and $1, 2, \dots$, be the pendant vertices adjacent to the vertices in the path respectively.

Label the path vertices with the odd tribonacci numbers and label the pendant vertices with the even tribonacci numbers.

Then we get the edge labeling as $(1) = -1$ and $(0) =$ so that

$$|(0) - (1)| = -(-1) = 1. \text{ Therefore,}$$

comb graphs are TLC graphs.

Corollary: 2.6

- (i) Let $\cong P_n \odot K_1$, $n = 2k$, $= 2, 3, \dots$. The addition of a pendant edge to a path vertex, that is labeled with odd tribonacci number, of G admits TL-cordial Labeling.
- (ii) Let $\cong P_n \odot K_1$, $n = 2k + 1$, $= 1, 2, 3, \dots$. The addition of a pendant edge to any vertex of G does not admit TL-cordial Labeling.

In the case of $n = 2k + 1$, $= 1, 2, 3, \dots$, the cardinality of even T_n 's is greater than that of odd T_n 's resulting in the absolute difference greater than 1 and hence G does not admit TL-cordial Labeling.

Theorem: 2.7

Star graphs 1_n , are TLC graphs except for $= 4 + 2$, $= 1, 2, 3, \dots$

Proof:

Let $1, 2, \dots$, be the vertices of star graph and let it has edges.

Let be the central vertex and $1, 2, \dots, -1$ be the pendant vertices of the star 1_n .

Label the central vertex by 1 (i.e.,) 1 and label the pendant vertices by the succeeding T_n 's.

Case (i): $n = 2 + 1$ for $n = 1, 2, 3, \dots$

As $n = 2 + 1$, there are even number of vertices in G .

Label the central vertex v_1 (i.e.,) 1 and label the pendant vertices by the succeeding

T_n 's so that the absolute difference of the number of zero's and the number of one's, differ

by 1. Hence $|f(0) - f(1)| = 1$.

Therefore, star graphs admit TL-cordial Labeling for $n = 2 + 1$ for $n = 1, 2, 3, \dots$.

Case (ii): $n = 4$ for $n = 1, 2, 3, \dots$

As $n = 4$, there are odd number of vertices in G .

Label the vertices as in the previous case so that the edge labeling results as the number

of zero's equal to number of one's. Hence $|f(0) - f(1)| = 0$.

Therefore, star graphs admit TL-cordial Labeling for $n = 4$ for $n = 1, 2, 3, \dots$.

Corollary: 2.8

Star graphs G_n do not admit TL-cordial Labeling for $n = 4 + 2, n = 1, 2, 3, \dots$

The result does not exist because in case of $q = 4n + 2$, the cardinality of even T_n 's is greater than the cardinality of odd T_n 's resulting in an absolute difference greater than 1.

s Corollary: 2.9

- (i) Let $G \cong G_n$, $q = 4n - 1$, $n = 1, 2, 3, \dots$. The addition of a pendant edge to any vertex of G admits TL-cordial Labeling.
- (ii) Let $G \cong G_n$, $q = 4n$, $n = 1, 2, 3, \dots$. The addition of a pendant edge to any vertex of G admits TL-cordial Labeling.
- (iii) Let $G \cong G_n$, $q = 4n + 1$, $n = 1, 2, 3, \dots$. The addition of a pendant edge to any vertex of G does not admit TL-cordial Labeling.

In the case of $q = 4n + 1$, the cardinality of even T_n 's is greater than that of odd T_n 's resulting in an absolute difference greater than 1 and hence G does not admit TL-cordial Labeling.

Theorem: 2.10

Bistar graphs, B_n , $n \geq 2$ admit TL-cordial Labeling.

Proof:

Let the bistar graph B_n be a $(2 + 2, 2 + 1)$ graph. Let y be the edge joining the two stars G_1 and G_2 .

Let $v_1, v_2, \dots, v_{n-1}$ be the vertices adjacent to v_n and $v_{n+1}, v_{n+2}, \dots, v_{2n}$ be the vertices adjacent to v_{2n+1} .

Label v_n and y by 1 which contributes one '0' to $f(0)$. Label the vertices

$v_1, v_2, \dots, v_{n-1}$ by the odd T_n 's and $v_{n+1}, v_{n+2}, \dots, v_{2n}$ by the even T_n 's so that it contributes 'm' 0's and 'm' 1's which implies $f(0) = m + 1$ and $f(1) = m$.

Hence $|f(0) - f(1)| = 1$ providing B_n a TLC graph.

Corollary: 2.11

- (i) Let $\cong, n = 2 + 1, = 1, 2, 3, \dots$ The addition of a pendant edge to vertex 'x' of G admits TL-cordial Labeling.
- (ii) Let $\cong, n = 2, = 1, 2, 3, \dots$. The addition of a pendant edge to any vertex of G does not admit TL-cordial Labeling.

The result does not exist because in case of $n = 2$ in G, the cardinality of even T_n 's is

greater than the cardinality of odd T_n 's resulting in an absolute difference greater than 1.

Theorem: 2.12

Y- trees [1] are TLC graphs except for $n = 4 + 1, = 1, 2, 3, \dots$

Proof:

Let Y- tree, be a $(n + 2, + 1)$ graph with $(V = \{1, 2, \dots, n+1, +2\})$
and $O = \{+1, = 1, 2, 3, \dots (-1)\} \cup \{+1\} \cup \{+2\}$.

Define : $O \rightarrow \{1, 2, \dots +2\}$

Case(i): $n = 2$ for $n = 2, 3, \dots$

Assign even T_n 's to first $n + 1$ vertices of O and odd T_n 's to the remaining vertices contributing ' $n + 1$ ' 0's and ' n ' 1's that yields $|O - (1)| = 1$.

Hence are TLC graphs for $n = 2$ for $n = 2, 3, \dots$

Case(ii): $n = 4 - 1$ for $n = 1, 2, 3, \dots$

Assign even T_n 's to first 2 vertices of O and odd T_n 's to the remaining $2 + 1$ vertices

contributing ' 2 ' 0's and ' $2 + 1$ ' 1's that yields $|O - (1)| = 0$.

Hence are TLC graphs for $n = 4 - 1$ for $n = 1, 2, 3, \dots$

Corollary: 2.13

Y- trees do not admit TL-cordial Labeling for $n = 4 + 1, k = 1, 2, 3, \dots$

The result does not exist because in case of $n = 4k + 1$, the cardinality of even T_n 's is greater than the cardinality of odd T_n 's resulting in an absolute difference greater than 1.

Corollary: 2.14

- (i) Let $\cong, n = 4k + 2, = 1, 2, 3, \dots$ The addition of a pendant edge to a pendant vertex, which is labeled with odd tribonacci number, of G admits TL-cordial Labeling.
- (ii) Let $\cong, n = 4k - 1, = 1, 2, 3, \dots$. The addition of a pendant edge to any vertex of G admits TL-cordial Labeling.
- (iii) Let $\cong, n = 4k \& 4 + 1, = 1, 2, 3, \dots$. Addition of a pendant edge to any vertex of G does not admit TL-cordial Labeling.

In the case of $n = 4k \& 4 + 1$, the cardinality of even T_n 's is greater than that of odd T_n 's resulting in an absolute difference greater than 1 and hence G does not admit TL-cordial Labeling.

Theorem: 2.15

Twig graphs, $n \geq 3$ admit TL-cordial Labeling for $n \neq 4 + 1, = 1, 2, 3, \dots$.

Proof:

Let the Twig graph be a $(3 - 4, 3 - 5)$ graph.

Let $1, 2, \dots$ be the vertices of the path; $1', 2', \dots, -2'$ be the pendant vertices attached above the internal vertices of the path and $1'', 2'', \dots, -2''$ be the pendant vertices attached below the internal vertices of the path.

Define: $\phi \rightarrow \{1, 2, \dots, 3-4\}$.

If $n = 3$, G is isomorphic to K_4 which is a TLC graph. (Refer case(ii) of Theorem:2.4).

Case(i): $n = 2 + 2, n = 1, 2, 3, \dots$

- ❖ Label the vertices $1, 2, \dots$ by the odd T_n 's in the successive order so that it contributes $(-1)1'$ to $\phi(1)$.
- ❖ Label the vertices $1', 2', \dots, -2'$ by the first (-2) even T_n 's so that it contributes $(-2)0'$ to $\phi(0)$.
- ❖ Label the vertices $1'', 2'', \dots, -2''$ by the remaining T_n 's in the order so that it contributes $\frac{-4}{2} 0'$ and $\frac{-4}{2} 1'$.

$$\text{Now, } \phi(0) = -2 + \frac{-4}{2} = \frac{3-4}{2} \text{ and } \phi(1) = -1 + \frac{-4}{2} = \frac{3-6}{2}$$

$$\text{and we get } |\phi(0) - \phi(1)| = \left| \frac{3-4}{2} - \frac{3-6}{2} \right| = 1.$$

Therefore, Twig graphs admit TL-cordial Labeling for $n = 2 + 2, n = 1, 2, 3, \dots$.

Case(ii): $n = 4 + 3, n = 1, 2, 3, \dots$

- ❖ Label the vertices $1, 2, \dots$ by the odd T_n 's in the successive order so that it contributes (-1) one's to $\phi(1)$.
- ❖ Label the vertices $1', 2', \dots, -2'$ by the first -2 even T_n 's so that it contributes (-2) zero's to $\phi(0)$.
- ❖ Label the vertices $1'', 2'', \dots, -2''$ by the remaining T_n 's in the successive order so that it contributes $\frac{-2}{2}$ zero's and $\frac{-2}{2}$ one's.

$$\text{Now, } \phi(0) = -2 + \frac{-2}{2} = \frac{3-4}{2} \text{ and } \phi(1) = -1 + \frac{-2}{2} = \frac{3-4}{2}$$

$$\text{and we get } |\phi(0) - \phi(1)| = \left| \frac{3-4}{2} - \frac{3-4}{2} \right| = 0.$$

Therefore, Twig graphs admit TL-cordial Labeling for $n = 4 + 3, n = 1, 2, 3, \dots$

Corollary: 2.16

Twig graphs, $n \geq 3$ do not admit TL-cordial Labeling for $n = 4 + 1, n = 1, 2, 3, \dots$

The result does not exist because in case of $n = 4k + 1$, the cardinality of even T_n 's is greater than the cardinality of odd T_n 's resulting in an absolute difference greater than 1.

Corollary: 2.17s

- (i) Let $n \geq 3, n = 4, n = 1, 2, 3, \dots$ The addition of a pendant edge to any

vertex labeled with odd T_n 's of G admits TL-cordial Labeling.

(ii) Let $n \geq 3$, $n = 4 + 3$, $n = 1, 2, 3, \dots$. The addition of a pendant edge to any vertex of G admits TL-cordial Labeling.

(iii) Let $n \geq 3$, $n = 4 + 2$, $n = 1, 2, 3, \dots$. The addition of a pendant edge to any vertex of G does not admit TL-cordial Labeling.

In the case of $n = 4 + 2$, the cardinality of even T_n 's is greater than that of odd T_n 's resulting in an absolute difference greater than 1 and hence G does not admit TL-cordial Labeling.

Definition: 2.18

The H- graph of a path, denoted by $H(P_n)$ is the graph obtained from two copies of P_n with vertices $1, 2, \dots, n$ and $1, 2, \dots, n$, by joining the vertices $\frac{n}{2} + 1$ and $\frac{n}{2} + 1$, if n is odd and the vertices $\frac{n}{2}$ and $\frac{n}{2} + 1$, if n is even.

Theorem: 2.19

$H(P_n)$, $n \geq 3$ are TLC graphs.

Proof:

Let the H-graph be a $(2, 2 - 1)$ graph. Let $O = 1, 2, \dots, n$, and

$$O = \{1, 2, 3, \dots, n-1\} \cup \{1, 2, 3, \dots, n-1\} \cup \left\{ \frac{n}{2} + 1 \right\} \text{ if } n \text{ is odd}$$

and

$$O = \{1, 2, 3, \dots, n-1\} \cup \{1, 2, 3, \dots, n-1\} \cup \left\{ \frac{n}{2}, \frac{n}{2} + 1 \right\} \text{ if } n \text{ is even.}$$

even.

Label the vertices $1, 2, \dots, n$ by the even T_n 's in the order so that it contributes $(-1)^0$ zero's

to (0) and label the vertices $1, 2, \dots, n$ by the odd T_n 's in the order so that it contributes $(-1)^1$ one's to (1) .

As $1, 2, 3, \dots, n$ are labeled with even T_n 's and $1, 2, 3, \dots, n$ are labeled with odd T_n 's, both the edges $\frac{n}{2} + 1$ or $\frac{n}{2}$ induces

one zero to (0) in both the cases (i.e.,) either the value of n is odd or even.

Now $(0) = -1 + 1 = 0$ and $(1) = -1$ and we get $(0) - (1) = 1$

$$1 = -(-1) = 1$$

Therefore, $n \geq 3$ are TLC graphs.

Corollary: 2.20

Let $\cong, \geq 3$. The addition of a pendant edge to any vertex labeled with odd T_n 's of G admits TL-cordial Labeling.

Definition: 2.21

A coconut tree $CT(m, n)$ is the graph obtained from the path P_m by appending new pendant edges at an end vertex of P_m .

Theorem: 2.22

Coconut tree $CT(m, n)$, are TLC graphs except for $+ = 4 + 3, = 1, 2, 3, \dots$

Proof:

Let the Coconut tree $CT(m, n)$ be a $(+, + - 1)$ graph.

Let $1, 2, \dots$ be the vertices of the path P_m and $1, 2, \dots$ be the vertices appended to the end vertex of the path P_m and the edges are

$$12, 23, \dots, -1, 1, 2, \dots.$$

Label the end vertex of the path by T_1 (i.e., 1) and the pendant vertices $1, 2, \dots$ by the

odd T_n 's succeeding 1. If all the pendant vertices got labeled with the odd T_n 's, then label the

remaining path vertices by the even T_n 's.

If suppose, all the pendant vertices do not get labeled with the odd T_n 's, then label all the remaining vertices (i.e., both unlabeled pendant vertices and path vertices by the even T_n 's.

$$\text{Now, we get either } |(0) - (1)| = 0 \text{ or } |(p) - (1)| = 1.$$

Therefore, $CT(m, n)$ are TLC graphs except for $+ = 4 + 3, = 1, 2, 3, \dots$

Corollary: 2.23

$CT(m, n)$ do not admit TL-cordial Labeling for $+ = 4 + 3, = 1, 2, 3, \dots$

In the case of $+ = 4 + 3$, the cardinality of even T_n 's is greater than that of odd T_n 's

resulting in an absolute difference greater than 1 and hence G does not admit TLC labeling

Corollary: 2.17

Let $\cong CT(m, n)$, The addition of a pendant edge to any vertex of G labeled with odd T_n 's

admits TL-cordial Labeling.

Conclusion:

In this work, we proved the existence of TL-cordial Labeling of some standard graphs. Proceeding this, we will work to prove the same for some special classes of graphs.

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