

METHODOLOGY OF TEACHING THE "THEORY OF PROBABILITY AND ELEMENTS OF MATHEMATICAL STATISTICS" MODULE IN THE SCHOOL MATHEMATICS CURRICULUM

Murod Barakayev¹, Ahmadjon Axlimirzayev², Abduvali Shamshiyev³, Bakhshillo Olimov⁴

¹Professor, PhD, Tashkent State Pedagogical University, named after Nizami, Tashkent, Uzbekistan

²Professor, PhD, Andijan State University, Andijan, Uzbekistan

³Associate Professor, PhD, Jizzakh State Pedagogical University, Jizzakh, Uzbekistan

⁴Professor, PhD, Republic Pedagogical Research Institute named after Qori Niyazi, Tashkent, Uzbekistan

DOI: 10.63001/tbs.2025.v20.i04.pp523-532

KEYWORDS:

Stochastic, teaching, methodology, theory of probability, mathematical statistics, elements of combinatorics, statistical data, probability, event, incident, random event, certain event, impossible event, relationships between events, one event leads to another, event result, event as part of an event, equivalent events.

Received on:

16-09-2025

Accepted on:

12-10-2025

Published on:

21-11-2025

ABSTRACT

The article is dedicated to the methodology of teaching the "Theory of Probability and Elements of Mathematical Statistics" module in the school mathematics curriculum. It discusses the role of the subject "Stochastic" in the study of various fields, such as physics, chemistry, biology, geology, economics, linguistics, psychology, and sociology. The article also provides a brief history of the "Theory of Probability and Mathematical Statistics" course, the mathematical apparatus of probability theory, elements of combinatorics, topics covered in the systematic course "Theory of Probability and Mathematical Statistics, Elements of Combinatorics" taught in schools, as well as concepts such as sets and combinatorics, statistical data, probability, event, incident, random event, certain event, impossible event, relationships between events, how one event leads to another, event result, event as part of an event, and equivalent events. The methodology of teaching these concepts in schools is also discussed.

From the 1970s to the present day, "Stochastic," one of the main branches of "Applied Mathematics," has been defining the field of human activity. The subject "Stochastic," which combines the "Theory of Probability and Elements of Mathematical Statistics," is widely used across almost all areas of knowledge, including physics, chemistry, biology, geology, economics,

linguistics, psychology, sociology, and others.

Information related to the theory of probability appeared in the works of mathematical scientists who lived in the 17th-18th centuries. Mathematicians such as P. Laplace, B. Pascal, J. Bernoulli, and others have substantiated that meaningful decisions in various fields of human activity, including

social, cultural, educational, and scientific-production areas, cannot be made without using the methods mentioned above. This highlights the relevance of studying the "Theory of Probability and Mathematical Statistics" in the school mathematics curriculum.

Analysis of the content of school mathematics education in the 20th century shows that, after the educational reforms in the former socialist countries in the 1960s, the school textbooks created by academician A.N. Kolmogorov included information on combinatorics. It is known that the development of the "Theory of Probability and Mathematical Statistics" led to the emergence of a set of mathematical methods called stochastic methods.

The subject "Stochastic" studies problems related to dividing finite or specifically bounded sets into subsets, particularly those related to combinatorial structures. The branch of mathematics that deals with the arrangements of certain objects in accordance with specific rules and the methods of counting these arrangements is called combinatorics.

From the perspective of set theory, combinatorics is the branch of mathematics that studies combinations and sets, their unions, intersections, various combinations, and the organization of subsets using different methods.

In today's modern context, combinatorics is widely applied in many fields of human activity. For example, in physics, biology, chemistry, linguistics, economics, and modern information technologies, combinatorial problems are indispensable. In these professional fields, it is impossible to carry out effective work without skills and knowledge in solving practical combinatorial problems. It is well

known that elements of combinatorics are part of the "Theory of Probability and Statistics." This subject is widely used not only in natural sciences but also in the study of social and humanitarian sciences, as well as in agriculture and all sectors of industry. The above shows that without studying the "Theory of Probability and Mathematical Statistics, Elements of Combinatorics" in the school mathematics curriculum, it is impossible to train specialists who are capable of adapting to market relations. Therefore, one of the key tasks of modern mathematics education is to ensure that school graduates acquire sufficient knowledge, skills, and competencies in "Probability Theory, Mathematical Statistics, and Elements of Combinatorics."

In general, it is advisable to teach the following in the school mathematics curriculum in the section "Theory of Probability and Mathematical Statistics, Elements of Combinatorics":

On "Elements of Combinatorics": solving combinatorial problems based on counting variants, the multiplication rule.

On "Theory of Probability and Mathematical Statistics" (tables, diagrams, graphs, data in the form of average values of measurements): random events, their frequency, probability, events that can occur with equal probability, calculating their probabilities, ideas about geometric probability.

Additionally, students will be introduced to the concept and problems related to random events. As a result, students should be able to:

Understand and solve problems related to random events.

Work with diagrams, graphs, and tables.

Calculate average values of measurement results.

Solve combinatorial problems based on systematic listing of variants.

Use the multiplication rule to find the frequency and probability of random events in simple situations.

Based on the above, the main objectives of studying the course "Theory of Probability and Mathematical Statistics, Elements of Combinatorics" are as follows:

Students should be able to apply their knowledge of the theory of probability and mathematical statistics as a tool for describing real-world stochastic models of events.

Develop and improve skills in "practical" thinking regarding the probabilistic and statistical aspects of solving problems in probability theory and mathematical statistics.

Use the apparatus of probability theory to enhance the level of mathematical culture of students.

Prepare students for further study in this field in higher education, and more.

It is important to emphasize that the mathematical apparatus of probability theory must be directed towards elementary mathematical knowledge and practical activities that students need to develop. In this process, students should:

Be able to perform arithmetic operations with real numbers.

Be able to correctly and appropriately use functional notations.

Have a certain level of knowledge, skills, and competence to work with simple geometric objects.

In the general secondary education mathematics curriculum, it is essential that students acquire the knowledge, skills, and abilities to solve combinatorial problems

using formulas for permutations, combinations, and groupings, and to study the Newton binomial formula and the properties of binomial coefficients. Additionally, students should study elementary and complex events, calculate the probabilities of events and their complements, and solve related problems.

It is worth noting again that the mathematical apparatus of probability theory is based on elementary mathematical knowledge and the practical experience that needs to be developed in students. During the study of "Probability Theory and Mathematical Statistics, Elements of Combinatorics," students should be able to perform arithmetic operations on real numbers, use functional notations correctly, have concepts about simple geometric figures, and be capable of working with them. Moreover, during this period, students will acquire sufficient knowledge about sets, which is crucial for understanding probability theory as a branch of mathematics. Through the study of applied theories, students will have the opportunity to acquire important knowledge for practical human activities.

Currently, one of the key aspects of modernizing the content of mathematical education is the integration of probability theory, statistics, and elements of combinatorics into the general secondary education curriculum, due to the necessity of acquiring knowledge of probability, statistics, and combinatorics in contemporary education. Without adequate knowledge of these fields, it is challenging to properly perceive social, political, and economic data and make decisions based on them. Especially in modern sciences such as physics, chemistry, biology, and the entire social-economic science domain,

probability-statistical foundations are being developed.

Although stochastic content has already been included in textbooks for classes specializing in deeper mathematics, it has not been organized into a coherent system, and this does not help develop a holistic approach to studying these problems.

The inclusion of probability theory, statistics, and elements of combinatorics into the general secondary education mathematics curriculum requires extensive research into teaching methods for these subjects. In the mid-20th century, the necessity of incorporating the simplest concepts of probability theory into school mathematics courses was substantiated by academician A. N. Kolmogorov.

According to him, the study of combinatorics, "Probability Theory and Mathematical Statistics, Elements of Combinatorics" in a systematic course should include the following:

Definitions, proofs, axioms, and theorems; necessary and sufficient conditions; contradiction theorems and proving the opposite of a theorem; direct and reverse theorems; understanding of axiomatic construction and "Geometry" as an axiomatic system, including Euclid's fifth postulate and its history.

Sets and combinatorics: Elements of a set, subsets, union, and intersection of sets, Euler diagrams. Examples of combinatorial problems: counting variants, multiplication rule.

Statistical data: Presentation of data in tables, diagrams, and graphs; average values; statistical conclusions based on a sample; understanding random events and examples.

Probability: Frequency of an event, probability, equivalent events and calculating

their probability, and introducing geometric probability.

It is known that the requirements for each school graduate during their study of mathematics include the ability to:

Perform simple proofs.

Draw simple conclusions based on previously known or derived statements and assess their logical correctness.

Use counterexamples and visual methods to refute statements.

Identify necessary information from data presented in tables, diagrams, and graphs.

Construct tables, diagrams, and graphs based on given data.

Solve combinatorial problems by systematically choosing possible variants and applying the multiplication rule.

Calculate the average values of measurement results.

Determine the frequency of an event based on personal observations and statistical data.

Find the probability of simple random events.

Apply knowledge, skills, and competencies in combinatorics, statistics, and probability theory in practical activities and daily life.

Identify necessary proofs in logical arguments (both monologue and dialogue formats).

Distinguish logically incorrect ideas.

Write mathematical proofs and statements based on logical operations.

Analyze numerical data presented in tables, diagrams, or graphs.

Solve practical problems involving numbers, percentages, lengths, areas, volumes, time, speed, etc., in daily and professional activities.

Solve educational and practical problems that require systematic selection of variants.

Evaluate the probability of random events occurring in practical situations and compare this with real-life models, understanding statistical conclusions.

In today's modern society, each individual is expected to:

Analyze random factors.

Assess their capabilities.

Propose various hypotheses.

Predict the development of a given situation.

Make correct decisions in situations with probabilistic characteristics or uncertainty.

The formation of these competencies and skills plays an important role in the study of the "Probability Theory and Mathematical Statistics, Elements of Combinatorics" course.

One of the key aspects of modernizing school mathematics education in the 21st century is the integration of "Probability Theory and Mathematical Statistics, Elements of Combinatorics" into the general secondary education curriculum. Integrating probability theory and mathematical statistics:

Helps form the concepts of determinism and randomness.

Identifies that many laws of nature and society have probabilistic characteristics.

Allows us to understand that real-world events and processes are described by probability models.

The teaching of the "Probability Theory and Statistics" module can be illustrated with the following example:

You bought a lottery ticket. You might win or lose. These are considered random events.

Random events may occur in different ways under similar conditions. For example, a candidate may win an election, or another candidate may win.

Let's examine the types of random events:

For example, when tossing a coin, either the head or the tail may appear. The probability of these events is equal. These are called equally likely events.

However, not all events may occur under the same conditions. For example, an alarm clock may ring, or it may not. A bus might stop running, or the light may burn out. However, under normal conditions, these events are unlikely.

Events that are impossible to happen are called impossible events. For example, when the power supply is disconnected, the light cannot stay on.

It is important to understand whether an event in the world of randomness can happen or not. How can we assess the probability of an event occurring? The answers to these questions are provided by the "Probability Theory and Statistics" subject.

Now, let's review some examples of random events.

.

Problem 1:

Identify which of the following events is random, certain, or impossible.

A turtle learns to talk – Impossible event;

Water in a pot on the stove boils – Certain event;

Your birthday is June 10 – Random event;

Your friend's birthday – Impossible event;

You participate in the lottery and win – Random event;

You participate in the lottery and do not win – Random event;

You lose a chess game – Random event;

Tomorrow you meet a traveler – Random event;

The weather worsens next week – Random event;

Today is not Thursday – Random event;

After Thursday comes Friday – Certain event;

After Friday comes Thursday – Impossible event.

One of the key elements that helps to shape the concept of an "event" is classifying events based on their "objective possibility of occurrence." Based on this, studying the classification of events is of significant philosophical importance for students.

In probability theory, the concept of "event" is closely linked with set theory concepts. Specifically, according to the definition of an event, it represents a set of elementary outcomes. Therefore, in order to properly introduce this concept, students need to familiarize themselves with the established theoretical elements of probability theory.

Analyses show that in the school mathematics curriculum, this theory is not sufficiently covered. This is why it is important for educators to develop methodological guidelines and recommendations on how to address this issue.

The methodology for studying probability theory materials largely resembles the approach introduced in the 1960s-70s by academician A. N. Kolmogorov in the textbooks for grades IV-V. These publications are preferable for studying the fundamental concepts of

probability theory, as they present logically defined paths based on essential concepts that serve as the foundation for introducing the basic concepts of probability theory.

The concept of "event" presents psychological difficulties for students. They tend to associate it with certain isolated internal actions.

According to the definition of the "event" concept, in addition to a single internal action, it is necessary to consider some that require more thought, and the number of elements involved will either be greater or equal. Moreover, in the concept of "event," it is important to clearly differentiate the "experiment" concept as the outcome under certain conditions.

For students, the concepts of "experiment" and "event" often coincide.

To help students form the idea of events, it is advisable to start by looking at the simplest probability models, such as tossing a die, drawing balls from a box, drawing cards from a deck, shooting at a target at an intuitive level, etc. Additionally, introducing the historical context of the main concepts is logically correct, as it does not disrupt the logical connection in the study of probability theory materials (for example, examining Bayes' experiment, where in each trial, different, incompatible outcomes can occur. Each of these outcomes is called an elementary event or elementary outcome. Based on these trials, the concept of a "complete set of events" can be introduced, which is a set of elementary events that are mutually incompatible. All of these concepts help to form the definition of the concept of "event" and serve to establish the fundamental ideas of this important concept in probability theory).

One more element that contributes to shaping the concept of "event" is the

classification of events based on the "objective possibility of occurrence," which is essential in developing a significant worldview in students.

It is known that in the world around us, there are only reliable, impossible, and random events. The core characteristic of the "random" event concept is explained through the study of probability models of reality. Examples of such models are naturally drawn from school subjects (physics, chemistry, geography, biology, history, social sciences, economics), which also fosters interdisciplinary connections.

The importance of classification based on the above characteristic lies in the fact that it forms the first approach to shaping the concept of "probability." If you try to compare the likelihood or impossibility of a specific event with a certain numerical measurement, especially by assigning a number corresponding to each reliable event and each impossible event, it becomes clear that each random event corresponds to a real number within an interval. At this point, it logically follows to study operations on events without temporarily revealing the issue of establishing a correspondence between random events.

First, the concept of "relations between events" must be considered, and it is logically appropriate to introduce the concept of "equivalent events" based on relationships such as "an event causes another event," "an event is the result of another event," or "an event is part of another event." When studying operations on events, it is essential to use visual and graphical tools from probability theory. Operations on events such as addition (union) and multiplication (intersection) are studied. The difference between events (subtraction) can be introduced through the corresponding

definition or based on the introduced operations. Here, it is emphasized that events are sets. Using examples based on basic probability models, operations on events can be studied in a manner similar to operations performed on sets.

Having multiple theoretical concepts in probability theory allows students to observe the complete similarity between operations on events. Probability theory shows students that the objects of relations in this section of mathematics are essentially the same as set theory, with the only difference being the terminology and language used in probability theory. It is helpful to provide students with a correspondence table between the terms of probability theory and set theory.

It is worth noting that developing the necessary skills and using the key properties of operations on events during the learning process will play a significant role in solving problems that may arise during the study of probability theory.

One of the most important problems discussed in probability theory is determining the probability of complex events obtained from simple events through operations. Furthermore, studying operations on events is crucial in cases where the probability space has many elements, as it leads to difficult calculations when solving problems. These rules can serve as the basis for motivating the study of operations on events.

When studying operations on events, it is appropriate to not only focus on the essence of the operation being studied but also to present problems that clearly reflect the difference between these operations. Typically, students easily determine the union and intersection of events. However, forming a deep understanding of the essence of operations on events may pose some challenges for students.

One of the main methodological challenges when studying the elements of probability theory is the process of solving problems related to simple events. This problem can be resolved based on the experience accumulated from solving other problems.

Studying operations on events should lead students to a deeper understanding of concepts such as "sample space of elementary events," "mutually exclusive events," "certain events," "impossible events," and "complementary events," as these concepts can be defined through operations on events.

After studying operations on events, the elements of combinatorics are also studied. The methodology for teaching the elements of combinatorics in school mathematics is not sufficiently developed. Currently, studying the elements of combinatorics is crucial as it plays an important role in various fields of knowledge, including computer science, as part of discrete mathematics.

However, the elements of "Combinatorics" can only be effectively studied through the "Probability Theory" topic.

The second fundamental concept in probability theory is "probability," which serves as the foundation for constructing all probability schemes describing a wide class of "random events." The formation of this "probability" concept also helps overcome the contradiction between the mathematical definition of the "event" concept and the subjective experience of students using the term "probability" in everyday practice.

Examples for determining the probability of events based on the classical definition of probability include problems such as the probability of tossing an

asymmetric coin, the probability of having a boy or a girl, or the likelihood of a "head" or "tail" appearing.

The formation of the "probability" concept can take place in several stages. First, considering the historical approach, the classical definition of probability is reviewed.

The probability of an event is the ratio of the number of favorable outcomes to the total number of possible outcomes. This definition is constructive and provides a method for calculating the probability of events in classical experiments. If many events occur as a result of an experiment that meets the following conditions, the experiment is called a classical experiment:

All events have an equal likelihood of occurring;

They are mutually exclusive;

A complete set of events is formed.

Historically, such events were referred to as possibilities, situations, or outcomes. Previously discussed probability models, such as tossing dice, drawing balls from a box, drawing cards from a deck, and shooting at a target, are examples of classical experiments.

Thus, it can be checked that the definition of probability introduced has the following characteristics:

The probability of a certain event is equal to 1;

The probability of an impossible event is equal to 0, as probability values fall within the interval $[0, 1]$;

If events are mutually exclusive, the sum of their probabilities equals the probability of their union.

One of the significant drawbacks of the classical definition of probability is that it only applies to classical experiments, which are rarely encountered in everyday practice.

It is important for students to understand that the classical definition of probability serves a narrow class of events, and therefore, alternative approaches to determining probability should be explored.

After reviewing the methods and techniques for teaching probability theory, it is appropriate to answer the question of whether this topic is taught in school mathematics courses. Afterward, it is useful to review the methods for studying the main theorems of probability theory.

In general, organizing the teaching of the "Probability Theory and Mathematical Statistics" module in this way will ensure that students acquire a solid understanding of the material and enhance the practical applications of this module by strengthening its applied aspects.

References

1. Bunimovich E.A. On the Theory of Probability and Statistics in the School Curriculum. – Moscow: *Mathematics in School*, 2009, No. 7, pp. 3-13.
2. Barakayev M. Methods of Teaching Mathematics (Part II, Special Methodology, Textbook) – Tashkent: *GRANT KONDOR PRINI*, 2024, 282 pages.
3. Barakayev M., Shamshiyev A., Goyibnazarova G., Jalilov A., Berdyarov A. Possibilities for Improving Mathematics Teaching in Modern Educational Conditions. – *Journal of Pharmaceutical Negative Results*, Volume 13, Special Issue 8, 2022, pp. 2312-2318.
4. Bunimovich E.A. Probability-Statistical Line in the Basic School Mathematics Curriculum. – *Mathematics in School*, 2002, No. 4, pp. 52-58.
5. Barakayev M., Shamshiyev A. The Need for Using Practical Problems in Teaching Mathematics. – Jizzakh: *Tafakkur Ziyosi*, 2019/4, pp. 48-58.
6. Fedoseev V.N. Elements of Probability Theory for 9th Grade of Secondary School. – Moscow: *Mathematics in School*, 2002, No. 5, pp. 34-40.
7. Barakayev M., O'rinov H. Methodological Issues of Teaching the Module "Integral and Its Applications" in the General Secondary Education Mathematics Curriculum. – Tashkent: *PEDAGOGICA* (Scientific-Theoretical and Methodical Journal), 2019/6, pp. 78-86.
8. Tokmazov G.V. Consolidation of Didactic Units in Probability Theory Problems. – Moscow: *Mathematics in School*, 1999, No. 4, pp. 81-84.
9. Barakayev M., Baxromov S.A. The Main Goal of Studying "Elements of Probability Theory and Mathematical Statistics" in the School Mathematics Curriculum. – *Ministry of Higher and Secondary Special Education of the Republic of Uzbekistan, Fergana State University, Uzbekistan Academy of Sciences, V.I. Romanovskiy Institute of Mathematics – Abstracts of the International Scientific Conference on Modern Problems of Differential Equations and Related Branches of Mathematics*, Fergana, March 12-13, 2020, pp. 364-366.
10. Barakayev M., O'rinov H. Preparing Mathematics Teachers for Innovative Pedagogical Activities as a Factor for the Effective Teaching of the "Elementary Mathematics" Course. – *Ministry of Higher and Secondary Special Education of the Republic of Uzbekistan, Fergana State University, Uzbekistan Academy of Sciences, V.I. Romanovskiy Institute of Mathematics – Abstracts of the International Scientific Conference on Modern Problems of Differential Equations and Related Branches*

of *Mathematics*, Fergana, March 12-13, 2020, pp. 331-333.

11. Potapova K.S., Vovnyanko A.A. Methodological Aspects of Teaching Statistics and Probability Theory in the School Mathematics Curriculum. – Moscow: *Young Scientist*, 2020, No. 49 (339), pp. 421-423.

12. Mordkovich A.G., Semenov P.V. On the Integration of the Stochastic Line in the Established School Mathematics Curriculum. – *Mathematics in School*, 2009, No. 7, pp. 38-45.

13. Motikas V.S. Probability Theory for Schoolchildren: A Textbook for an Elective Course for 8-10 Grade Students. – Moscow: *Prosveshchenie*, 1976, 104 pages.

14. Axlimirzayev A., Nishanov T. The Role and Importance of the Practical-Professional Approach to Teaching Probability Theory and Mathematical Statistics in the Training of Future Economists. – *Universum: Psychology and Education*, 2021, No. 2 (80), pp. 12-15.