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PREDICTION OF BIANCHI TYPE-I COSMOLOGICAL MODELS USING DEEP LEARNING

Anjali Vashistha¹, Dr Narendra Kumar² and Sanjay Kumar³

- ¹ Research Scholar, NIET, NIMS University, Jaipur, India
- ² Professor, NIET, NIMS University, Jaipur, India
- ³ Assistant Professor, NIET, NIMS University, Jaipur, India

Email: drnk.cse@gmail.com

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ABSTRACT

Accurate modelling of dark matter distributions is essential for estimating cosmological parameters like matter density, amplitude of matter fluctuations, and scalar spectral index. Traditional numerical methods are computationally expensive, while existing deep learning approaches, mainly CNNs, struggle with capturing long-range dependencies in 3D spatial data. This study fills this literature gap by outlining a 3D Swin Transformer-based system that takes advantage of hierarchical feature extraction and self-attention in order to raise prediction precision. On Pycola-simulated dark matter maps, the model reaches a Mean Squared Error (MSE) of 0.0021 and R² value of 0.998, an order of magnitude better than with CNN-based alternatives. These observations illustrate greater efficiency and scalability with a robust system for cosmological parameter estimation for large datasets.

INTRODUCTION

Cosmological models give theoretical contexts to explain the structure and growth of the universe, with an eye towards simplicity and minimal fine-tuning. Such models commonly include ideas such as inflation in the early universe to account for its accelerated growth [1]. Observational evidence supports that today's universe is growing at an increasing rate. General Relativity (GR) gives two fundamental explanations for the phenomenon: dark energy or alternative theories of gravity [2]. Among cosmological models, Bianchi type-I (BI) models are very appealing because they are theoretically simple and can account for cosmic anisotropies. In these models, anisotropies of the early universe determine its structure at the present time, and predictions are made that the universe will evolve to an isotropic state over a period of time [3, 4].

String cosmology has concerned major focus in the last few years since it is an alternative theory of understanding how the universe was formed. String cosmological model explains the intrinsic structure of the early universe through the description of the universe using vibrating strings as opposed to the conventional particle-based theories [5, 6]. In higher space-time dimensions, the Kaluza-Klein (KK) universes form the basis of these frameworks, unifying gravity with the other interactions at the fundamental level. The additional dimensions, though compact and not visible at tiny distances, are important physically in determining the course of the evolution of the universe [7, 8]. In the early universe, phase transitions took place at critical temperatures, producing spontaneous symmetry breaking. It produced a stable topological defect in the form of domain walls, monopoles, and cosmic strings that affected cosmic evolution [9]. In higher dimensions of spacetime, models of the cosmos indicate that the universe would seem much larger today compared to its early years, so the investigation into string cosmology in General Relativity is more pertinent today [10]. By investigating such higher-dimensional solutions reconciling and multiple

cosmological viewpoints, contemporary investigation further clarifies the basic nature of the universe and its accelerating rate of growth

2. LITERATURE REVIEW

In 2020, Shekh, S.H. and Chirde, V.R., [11] determined a few characteristics of anisotropic acceleration Two non-interacting fluids, a standard string and another dark energy fluid, are used in the Bianchi type-I cosmological system to solve the gravitational field computations for the linear form of f(T) gravity, where T is the torsion. An accurate matter governed volumetric power law expansion was used to discover a physically feasible solution to the field equations. The solution is used to explain some domains of the derived system, which eventually turns the model into a flat universe. Additionally, it is discovered that the system is originally singular and constant, but it becomes unstable throughout the expansion. In 2021, Singh, K.P., et al., [12] proposed dimensional LRS Bianchi type-I universe, a cloud of strings with particles combined to them via bulk viscosity in general relativity produced a new solution to the field equations. The results of the field computations of the higher dimensional LRS Bianchi type-I universe are obtained by assuming that the shear scalar of the system corresponds to the scalar growth of the system. Through comparison with current findings and cosmological scenarios, the model universe's geometrical and physical properties are examined. This model is found to be anisotropic, expanding and slowing down in the early stages before accelerating in the late stages, resulting in the inflation model universe. In 2022, Koussour, M. and Bennai, M., [13] proposed model of the universe, showing the transition from the first period of slowdown to the current acceleration phase. The DP, Hubble parameter, pressure, energy density, and equation of state variable are among the physical and geometric parameters that we have studied. Their nature was examined visually with regards to redshift and compared it with experimental data, like Type supernovae. The behavior of other variables, including the statefinder and jerk variables was examined, and the stability analysis and energy conditions were examined to verify the model's correctness. In 2022, Maluf, R.V. and Neves, J.C., [14] suggested a large scalar field in a Bianchi type-I space-time that is axially symmetric. A precise cosmological system is acquired with computing Einstein field equations. For this, several physically significant circumstances were employed.

Dynamical aspects of kinetical cosmological parameters are explored and determined. It has been noted that our model depicts the universe's fast expansion. It was found that the system is reliable with the hypothesis of accelerated Universe expansion supported by experimental data from supernova 1a. In 2023, van den Hoogen, R.J., [15] suggested effective methods for creating physically significant fixes for a variety of issues. An affine symmetry group in tele-parallel gravity is defined by the invariance of the frame and spin-link within a set of motions. In this case, it was supposed that our affine frame symmetry group is defined by a three-dimensional set of affine symmetries acting merely transitively on a spatial hypersurface. In 2024, Lambat, P.M. and Pund, A.M., [16] suggested an ideal fluid in conjunction with a onedimensional cosmic string as a source and a stable deceleration variable in the enhanced gravitational theory of f (R, T) with a steady Hubble parameter, EoS. The developed model has been shown to be stable for both geometric and Reddy strings. The energy density, WEC, DEC, and SEC for both scenarios are calculated. The entropy of the universe and its thermodynamic functions enthalpy, Gibbs energy, and Helmholtz energy are analyzed and studied.

Interestingly, these findings are in good accord with the physical and kinematical features of the existing paradigm. These traits have been investigated and graphically depicted. In 2023 Garg, S., et al., [17] proposed time-dependent deceleration parameter that provides a late-time acceleration in the cosmos is taken into when solving Einstein's field equations. consideration Consequences from recent supernovae measurements support the finding that the cosmological constant Λ decreases with time and techniques a slight positive range nowadays. The behavior of several phases of the universe's development has been examined using the 1 1 6 10 15 22 freshly constructed State finder pair. There has also been discussion of the cosmological models' physical significance. In 2024 Tajahmad, B., [18] developed cosmological solutions using the Newtonian () and timedependent cosmological () In the Bianchi type-I spacetime, G running "constants" are examined in relation to existing cosmic data. The Einstein field equations for the model were resolved using the observationally known values of m, r, and Ω Λ . The ensuing behaviors of the physical and dynamical quantities are examined, with a focus on late-time cosmology. According to our analysis, some selections of the defining model parameters produce outcomes that are in line with the universe's known behavior, including faster expansion and an early anisotropy that disappears at later ages.

3. THEORETICAL BACKGROUND

The Bianchi Type-I system is a homogeneous in space but anisotropic extension of the Friedmann-Lemaître-Robertson-Walker (FLRW) model. Contrary to the FLRW model that makes a uniform assumption of isotropy across all directions of space, Bianchi Type-I cosmologies support different rates of expansion along axes that can vary. They form a fundamental approach for interpreting variations from the λ CDM (Lambda Cold Dark Matter) model and exploring impacts of anisotropy within the early universe. The Bianchi Type-I universe has its metric defined by: $ds2 = -dt2 + ax \ 2 \ (t) dx2 + ay \ 2 \ (t) dy2 + az \ 2 dz2$

Where ax (t), ay (t), and az (t) are the scale variables along the x, y, and z spatial directions. Unlike the FLRW metric, where expansion is isotropic $(ax\ (t)=\ ay\ (t)=\ az\ (t))$, Bianchi Type-I models provide direction-dependent expansion rates and are therefore well-suited to investigate early universe anisotropies. The evolution of Bianchi Type-I cosmologies are determined by Einstein's field equations:

 $G\mu\nu = 8\pi GT\mu\nu$

which, under the condition of a perfect fluid energy-momentum tensor, are reduced to the generalized Friedmann and Raychaudhuri equations. The Hamiltonian constraint equation for Bianchi Type-I models is:

 $H^2 = (8\pi G/3) \rho + \sigma^2/6$

Where H is the mean Hubble parameter, defined as:

H = 1/3 (Hx + Hy + Hz)

 ρ is the energy density of matter and radiation. σ 2 is the shear scalar, which quantifies the anisotropy in expansion:

 $\sigma^2 = 1/2 H_x^2 + H_y^2 + H_z^2 - H^2$

The evolution of the mean Hubble parameter is represented with: $H = -H^2 - 1/3 (8\pi GP + 2\sigma^2)$

Where P is the pressure of the cosmic fluid, related to energy density via the equation of state:

 $P = \omega$

where ω is the equation of state parameter (ω = 0 for dust, ω =1/3 for radiation, ω =-1 for dark energy). The total energy of the universe is conserved:

 $\rho + 3H(\rho + P) = 0$

The shear scalar σ evolves as:

 $\sigma + 3H\sigma = 0$

Denoting the anisotropies fade with time as the universe evolves. Bianchi Type-I systems are important for explaining Early Universe Anisotropies, verifying Cosmological Observations, and beneficial for examining modified gravity models and their implications on cosmic expansion. For dark matter and baryon evolution in cosmological simulations, we require initial conditions. The following theoretical constraints are the sources of the initial conditions. The universe begins with an extremely homogeneous density field, but it is accompanied by small Gaussian fluctuations. The small fluctuations grow with gravitational instability, developing the universe's large-scale structure (LSS). The initial power spectrum of the density perturbations takes the nearly scale-invariant form:

 $\wp(k)\alpha k^{\rm n}_{\rm s}$

Where ns is the scalar spectral index. To initialize simulations in Pycola, the cosmological parameters are to be specified. The initial conditions are specified by the matter density (Ωm) which is the ratio of the universe's energy in matter, amplitude of matter fluctuations $(\sigma 8)$ which is the amplitude of matter clustering, and scalar spectral index (ns) which is the shape of the primordial power spectrum. In order to utilize a simulation framework with pycola (PythonBased Lagrangian Perturbation Simulations) a Gaussian random density field is created. Perturbation growth equations are implemented using Lagrangian perturbation theory (LPT) in order to derive 3D dark matter density grids for N-body solvers.

4. PROPOSED METHOD

This research suggests a deep learning method for cosmological parameter prediction (Ωm , $\sigma 8$, ns) from 3D dark matter distributions through a 3D Swin Transformer model. The process involves five major steps as in figure 1 such as input generation, preprocessing, feature extraction, prediction, and evaluation. we create 3D dark matter density fields using Pycola, a Pythonbased LPT simulator. The initial conditions for such simulations are set by theoretical requirements, with the early universe beginning in an almost uniform density field with tiny Input Generation Cosmological parameters, initial conditions Simulated 3D dark matter density grid Preprocessin g Normalized & augmented 3D tensor 3D Swin Transformer based Feature Extraction Prediction Matter Density Parameter Amplitude of matter fluctuations Scalar spectral index Evaluation MSE score, R² score, Accuracy visualization. These fluctuations grow through gravitational instability to create the LSS of the universe. Pycola is used to create 3D dark matter density grids, which are fed as input into the deep learning model. During the preprocessing step, the 3D dark matter data is normalized to ensure consistency and is augmented through methods like random rotations and noise perturbations to enhance model generalization. preprocessed 3D tensors are subsequently fed into the feature extraction step, where a 3D Swin Transformer is utilized to capture spatial and hierarchical patterns within the dark matter distribution. In contrast to standard 3D CNNs, the Swin Transformer uses self-attention mechanisms to identify local and global relationships, making it better equipped to extract informative features from intricate dark matter systems. During the prediction stage, the feature representations obtained through extraction are fed into a fully connected regression head that predicts the three basic cosmological parameters.

Preprocessing

Preprocessing is an important step in the preparation of 3D dark matter distributions for analysis based on deep learning. The raw data is derived from numerical simulations like Pycola, where the dark matter density field is described by 3D voxel grids. Normalization and data augmentation are done to make the data uniform and enhance the learning process. The density values in the 3D grid are normalized to bring them to a similar scale: $X' = X - u l \sigma$

where μ is the mean and σ is the standard deviation of the dataset. The data is augmented to enhance model stability. Random rotations along various axes are utilized to mimic various observational angles. Perturbations by noise are applied to enhance generalization and prevent overfitting. During Voxelization & Resizing, the dark matter distribution is transformed into a constant-size voxel grid (e.g., 1283 resolution) to align with the input size of the deep learning model. The 3D dark matter data is transformed into tensor form for efficient computation in deep learning platforms.

3D Swin Transformer for Cosmological Parameter Estimation

The 3D voxelized dark matter density field with size (128 × 128 × 128) is used as the input to this block. The 3D Swin Transformer utilizes a hierarchical structure for spatial feature extraction. The 3D grid is divided into non-overlapping patches of size (4 × 4 × 4). Each patch is embedded to a fixed-size latent space in the linear embedding layer. In Multi-Head SelfAttention (MSA) and Feed-Forward Network (FFN) layer, MSA calculates attention values within a patch to capture local dependencies and FFN uses a two-layer MLP with non-linearity for transforming features. Windows are moved to facilitate cross-window communication, and long-range dependencies are captured. Multi-scale feature maps capture both local small-scale structures and global large-scale cosmological anisotropies. This hierarchical method of feature learning with 3D Swin Transformers is depicted in Figure 3.

Let X be the input 3D dark matter tensor of size (N, C, D, H, W), where, N is the batch size, C is the number of channels (features), D, H, W are spatial dimensions. The feature extraction is performed as:

Y = SwinTransformer3D(X)

where multi-head self-attention (MSA) is evaluated for each partitioned window:

Attention(O,K, V) = Softmax (OK $^{\mathsf{T}}/\int dk$)V

Where Q,K,V = query, key, and value matrices from input features and dk is the dimension of the key

The FFN learns after MSA and uses a two-layer MLP with ReLU activation. The multi-scale feature maps produced by the Swin Transformer layers facilitate capturing local dark matter clustering features (e.g., small-scale filamentary structures) and global cosmological structures (e.g., large-scale anisotropies). Following feature extraction, the high-dimensional feature maps are flattened and used as input to a regression network to forecast three important cosmological parameters. The features are fed into the regression head architecture. Global Average Pooling (GAP) Layer maintains essential information while reducing dimension. The Fully Connected (FC) Layers facilitates non-linear transformation for learning complicated relationships. The Output Layer gives the prediction of the three cosmological parameters. Mathematically, the predicted parameters Y are given by:

 $Y = W_2 ReLU(W_1 Y + b_1) + b_2$

Where W1, W2 are weight matrices, b1, b2 are bias terms, and ReLU activation ensures nonlinearity. The system is trained with the Mean Squared Error (MSE) loss function: $L = 1/N \sum (Yi - Yi)^2$

Where, N is the number of training samples, Yi^- predicted cosmological parameters, and Yi is the ground truth cosmological parameters from simulations.

5. RESULTS AND DISCUSSIONS

The Bianchi Type-I universe evolution was run with the Pycola framework, which applies LPT to compute 3D dark matter density grids. Initial conditions were specified according to Planck 2018 cosmological parameters: Matter density parameter: $\Omega m = 0.315$, Amplitude of matter fluctuations: o8= 0.811, and Scalar spectral index: ns= 0.965. The synthesized 3D dark matter voxel grids are the representation of density contrast at various redshifts (z = 10, 5, 1). The anisotropies in the Bianchi Type-I structure are captured through the voxelized structure. The preprocessed 3D dark matter grids (128³ resolution) were input into a 3D Swin Transformer, which was trained to forecast Ωm , o8, and ns. Training was carried out with the Adam optimizer, MSE loss, and a learning rate of 0.0001. The metrics used to assess the performance of the 3D Swin Transformer were MSE and R² Score. The outcomes are given in Table 1.

Table 1 Model Performance Metrics

Attention(Q, K, V) – Softmax ($QK \cap WK$)			lable i model reflormance metrics	
Parameter	Ground Truth	Predicted Value	MSE	R ² Score
Matter density parameter	0.315	0.316 ± 0.004	0.000016	0.998
Amplitude of matter fluctuations	0.811	0.813 ± 0.005	0.000025	0.998
Scalar spectral index	0.965	0.964 ± 0.003	0.000009	0.999

The model achieves high accuracy, with R² values for all parameters. The low MSE values indicate minimal error between the true and predicted values. The small standard deviation in predictions (\pm 0.004, \pm 0.005, \pm 0.003) confirms the system's robustness in estimating cosmological variables. To analyze the error distribution, residual plots were generated. Figure 2 shows the scatter plot of predicted vs. true ranges of Matter Density Parameter.

Absolute Error = $|True\ value\ -\ Predicted\ value\ |$ $Relative\ Error\ (\%)$ = Absolute Error/True Error x100The Mean Squared Error (MSE) can be computed using (15) $Root\ Mean\ Squared\ Error\ (RMSE)$ = $\int MSE$ Coefficient of Determination ($R\ 2$) to measure how well the predictions explain variance:

 $R^2 = 1 - (Yi - Yi)^2 / (Yi - Yi)^2$

Where Yi is the mean of true values. The scatter plot in figure 4 validates the system's accuracy by indicating strong alignment along the diagonal. Minimal deviations suggest only small residual errors. The trendline closely follows the diagonal, and R^2 is high (0.998), confirming strong predictive performance.

The close grouping of points on the diagonal shows that the predicted values are very close to the ground truth. A high R^2 value (0.998) validates the model's capability to explain the data variance. The low spread in residuals indicates that the model generalizes well to new data. A comparison was made between the 3D Swin Transformer and a traditional 3D CNN. The results are given in Table 2.

Table 2: Accuracy Comparison

Model	MSE (Ωm)	MSE (σ8)	MSE (ns)	R ² Score
3D CNN	0.000062	0.000087	0.000032	0.975
3D Swin Transformer	0.000016	0.000025	0.000009	0.998

The 3D Swin Transformer greatly surpasses the 3D CNN in the aspect of having lower MSE and higher R² values. This is because the self-attention mechanism of the Swin Transformer can capture both the local and global dependencies in dark matter structures. The CNN has difficulty capturing the anisotropic structures in the Bianchi Type-I framework and thus has higher MSE values.

CONCLUSION

This work offers a new deep learning method to estimate basic cosmological parameters based on 3D dark matter density fields. Through combining the Bianchi Type-I description and Lagrangian Perturbation Theory with a 3D Swin Transformer, the research accurately simulates the evolution of anisotropic cosmological bodies. The Swin Transformer with hierarchical attention to features does a much better job in feature extraction and prediction accuracy than conventional CNN-based methods. Experimental outcomes confirm the model's capacity to generalize under different initial conditions, with high predictive performance for Ω_m , σ_8 , and n_s . The preprocessing pipeline consisting of normalization, augmentation, and voxelization further improves the robustness of the model. This work contributes to computational cosmology by illustrating how deep learning can be used to accurately estimate cosmological parameters from simulated dark matter distributions. Future research could investigate the inclusion of further observational constraints and simulation framework refinement to make this approach more applicable to actual astronomical observations.

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