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P-order *e*-open Continuous Mapping in Cubic Topological Spaces

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ABSTRACT

In this paper, we introduce a P-cubic e-continuous mapping in Porder cubic topological spaces. Also, we discuss about nearby open sets, their properties and examples of it. Moreover, we look into some of their primary properties and examples of P-cubic e-continuous in a P order cubic topological space.

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INTRODUCTION

The concept of fuzzy set and interval-valued fuzzy set (IVFS) was first proposed by Zadeh [14, 15]. Following this, fuzzy topological space was introduced by C. L. Chang [3] in 1968. Subsequently, in 2012, Y. B. Jun [9] utilized the notions of fuzzy sets and interval-valued fuzzy sets to introduce a novel set called cubic set. Akhtar [1], in 2016, constructed a topological structure based on cubic set theory, termed as cubic topological space, which discussed two variants known as P-cubic topological space and R-cubic topological space. Further advancements were made in 2019 by Loganayaki and Jayanthi [11], who introduced interior and closure in P-cubic topological space and R-cubic topological space, along with various types of open sets and continuous mappings on these spaces.

In a series of significant contributions, E. Ekici [4, 5, 6, 7, 8] extensively investigated the properties of e and e^⋆ sets, along with nearby open sets, within the context of general topological spaces. Ekici's research provided valuable insights into the behavior of these sets, contributing significantly to the understanding of topological structures.

The objective of our paper is to introduce P-order e-open Continuous Mapping and its associated nearby open sets. We aim to establish solid theorems and provide illustrative examples to support our propositions.

Preliminaries

Definition 2.1 *[15] A closed sub-interval of $I=[0,1]$ is called interval number. $a=[a^-, a^+]$ where $0\leq a^-\leq a^+\leq 1$. [I] denotes the set of all interval numbers.*

Definition 2.2 *[15] Let X be a non-empty set. A function $A: X\to [I]$, from X to all interval number is called interval valued fuzzy set (IVFS) in X. $[I]^X$ denotes the set of all IVFS in X. $\forall A\in [I]^X and x\in X A(x)=[A^-(x),\ A^+(x)] is$ called degree of membership of x in A. individually $A^-: X\to I$ and $A^+: X\to I$ is Fuzzy set in X. Simply A^- is called lower fuzzy set and A^+ is called upper fuzzy set.*

Definition 2.3 *[9] Let X be a non-empty set, Then a structure $A=\{\langle x,\ \mu(x),\ \lambda(x)\rangle / x\in X\}$ is cubic set in X in which μ is interval valued fuzzy set (IVFS) in X and λ is fuzzy set in X. Simply a cubic set is denoted by $A=\langle\mu,\lambda\rangle$ and C^X denotes the collection of all cubic sets in X.*

Definition 2.4 *[9] Let $X\neq\phi$, Then a cubic set $A=\langle\mu,\lambda\rangle$ is said to be internal cubic set (ICS) if $\mu^-(x)\leq\lambda(x)\leq\mu^+(x)\forall x\in X$.*

Definition 2.5 *[9] Let $X\neq\phi$, Then a cubic set $A=\langle\mu,\lambda\rangle$ is said to be an external cubic set (ECS) if $\lambda(x)\not\in(\mu^-(x),\ \mu^+(x))\forall x\in X$.*

1. A cubic set $A=\langle\mu,\lambda\rangle$ in which $\mu(x)=0$ and $\lambda(x)=1$ (resp. $\mu(x)=1$ and $\lambda(x)=0)\forall x\in X$ is denoted by $\ddot{0}$(resp.$\ddot{1}$) .
2. A cubic set $A=\langle\mu,\lambda\rangle$ in which $\mu(x)=0$ and $\lambda(x)=0$ (resp. $\mu(x)=1$ and $\lambda(x)=1\forall x\in X$ is denoted by $\hat{0}$(resp. $\hat{1}$) .

Let $A = \langle \mu, \lambda \rangle$ and $B = \langle \beta, \eta \rangle$ be two cubic sets in X, Then we define;

1. $A = B \Leftrightarrow \mu = \beta$ and $\lambda = \eta$
2. $A \subseteq_P B \Leftrightarrow \mu \subseteq \beta$ and $\lambda \leq \eta$
3. $A^c =< \mu^c,\ 1 - \lambda >= \{\langle x,\ \mu^c(x),\ 1 - \lambda(x) \rangle / x \in X\}$
4. $(A^c)^c = A$
5. $\hat{0}^c = \hat{1}$ and $\hat{1}^c = \hat{0}$
6. $(\cup_P A_i)^c = \cap_P A_i^c$ and $(\cap_P A_i)^c = \cup_P A_i^c$
7. P-Union $\cup_{i\in\mathbb{N}} A = \{\langle x,\ (\cup_{i\in\mathbb{N}} \mu_i)(x), (\vee \lambda_i) i \in \mathbb{N}(x) / x \in X \rangle\}$
8. P-Intersection $\cap_{i\in\mathbb{N}} A = \{\langle x,\ (\cap_{i\in\mathbb{N}} \mu_i)(x),\ i \in \mathbb{N}(\wedge \lambda_i)(x) / x \in X \rangle\}$

Definition 2.6 *[1] A P- cubic topology (in brief Pct) is the family $\mathcal{F}_P$ of cubic sets in X which satisfies the following conditions;*

1. $\hat{0}, \hat{1} \in \mathcal{F}_P$.
2. Let $A_i \in \mathcal{F}_P$, Then $\cup_P A_i \in \mathcal{F}_P . i \in \mathbb{N}$
3. Let $A, B \in \mathcal{F}_P$, Then $A \cap_P B \in \mathcal{F}_P$.

The pair $(X, \mathcal{F}_P)$ is called P-cubic topological space (in brief, Pcts).

Definition 2.7 *[11] A set R is said to be a P-order Cubic set (in brief, CS_P) [(i)]*

1. regular open set (briefly, CS_Pros) if $R = CS_Pint(CS_PclR)$.
2. regular closed set (briefly, CS_Prcs) if $R = CS_Pcl(CS_PintR)$.

Definition 2.8 *[11] A set R is said to be a CS_P [(i)]*

1. interior(resp. δ interior) of R (briefly, CS_PintR (resp. $CS_P\delta int$)) is defined by CS_PintR (resp. $CS_P\delta int$) $= \cup\ \{\tilde{G} : \tilde{G} \subseteq R\ \&\ \tilde{G}$ is a CS_Pos (resp. $CS_P\delta os$) in $X\}$.
2. closure(resp. δ closure) of R (briefly, CS_PclR (resp. $CS_P\delta cl$)) is defined by CS_PclR (resp. $CS_P\delta cl$) $= \cap\ \{\tilde{G} : \tilde{G} \supseteq R\ \&\ \tilde{G}$ is a CS_Pcs (resp. $CS_P\delta cs$) in $X\}$.

Definition 2.9 *[11] A set R is said to be a CS_P [(i)]*

1. β open set (briefly, $CS_P\beta os$) if $R \subseteq CS_Pcl(CS_Pint(CS_PclR))$.

Definition 2.10 *[12] A set R is said to be a CS_P [(i)]*

1. δ-pre open set (briefly, $CS_P\delta\mathcal{P}os$) if $R \subseteq CS_Pint(CS_P\delta clR)$.
2. δ-semi open set (briefly, $CS_P\delta\mathcal{S}os$) if $R \subseteq CS_Pcl(CS_P\delta intR)$.
3. e-open set (briefly, CS_Peos) if $R \subseteq CS_Pcl(CS_P\delta intR) \cup CS_Pint(CS_P\delta clR)$.
4. e^*-open set (briefly, CS_Pe^*os) if $R \subseteq CS_Pcl(CS_Pint(CS_P\delta clR))$.
5. a-open set (briefly, CS_Paos) if $R \subseteq CS_Pint(CS_Pcl(CS_P\delta intR))$.

The complement of a CS_Pe-open set (resp. $CS_P\delta os$, $CS_P\delta\mathcal{P}os$, $CS_P\delta\mathcal{S}os$ & CS_Pe^*os) is called a neutrosophic soft e- (resp. δ, δ-pre, δ-semi & e^*) closed set (briefly, CS_Pecs (resp. $CS_P\delta cs$ $CS_P\delta\mathcal{P}cs$, $CS_P\delta\mathcal{S}cs$ & CS_Pe^*cs)) in X.

The family of all $CS_P\delta\mathcal{P}os$ (resp. $CS_P\delta\mathcal{P}cs$, $CS_P\delta\mathcal{S}os$, $CS_P\delta\mathcal{S}cs$, CS_Peos, CS_Pecs CS_Pe^*os & CS_Pe^*cs) of X is denoted by $CS_P\delta\mathcal{P}OS(X)$ (resp. $CS_P\delta\mathcal{P}CS_P(X)$, $CS_P\delta\mathcal{S}OS(X)$, $CS_P\delta\mathcal{S}CS_P(X)$, $CS_PeOS(X)$, $CS_PeCS_P(X)$ $CS_Pe^*OS(X)$ & $CS_Pe^*CS_P(X)$).

Definition 2.11 *[12] A set R is said to be a CS_P [(i)]*

1. e interior(resp. δ *pre* interior & δ *semi* interior) of R (briefly, CS_PeintR (resp. $CS_P\delta\mathcal{P}int$ & $CS_P\delta\mathcal{S}int$)) is defined by CS_PeintR (resp. $CS_P\delta\mathcal{P}int$ & $CS_P\delta\mathcal{S}int$) $= \cup\ \{\tilde{G} : \tilde{G} \subseteq R\ \&\ \tilde{G}$ is a CS_Peos (resp. $CS_P\delta\mathcal{P}os$ & $CS_P\delta\mathcal{S}os$) in $X\}$.
2. e closure (resp. δ *pre* closure & δ *semi* closure) of R (briefly, CS_PeclR (resp. $CS_P\delta\mathcal{P}cl$ & $CS_P\delta\mathcal{S}cl$)) is defined by CS_PeclR (resp. $CS_P\delta\mathcal{P}cl$ & $CS_P\delta\mathcal{S}cl$) $= \cap\ \{\tilde{G} : R \subseteq \tilde{G}$ & R is a CS_Pecs (resp.$CS_P\delta\mathcal{P}cs$ & $CS_P\delta\mathcal{S}cs$) in $X\}$.

Definition 2.12 *[11] Let $(X, \mathcal{F}_P)$ and $(Y, \mathcal{G}_p)$ be any two $NSts$'s. A map $f : (X, \mathcal{F}_P) \to (Y, \mathcal{G}_p)$ is said to be CS_P [(i)]*

1. continuous (briefly, CS_PCts) if the inverse image of every CS_Pos in $(Y, \mathcal{G}_p)$ is a CS_Pos in $(X, \mathcal{F}_P)$.
2. β-continuous (briefly, $CS_P\beta Cts$) if the inverse image of every CS_Pos in $(Y, \mathcal{G}_p)$ is a $CS_P\beta os$ in $(X, \mathcal{F}_P)$.

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Definition 3.1 *Let $(X, \mathcal{F}_P)$ and $(Y, \mathcal{G}_p)$ be any two $NSts$'s. A map $f : (X, \mathcal{F}_P) \to (Y, \mathcal{G}_p)$ is said to be CS_P*

1. $\delta\mathcal{S}$-continuous (briefly, $CS_P\delta\mathcal{S}Cts$) if the inverse image of every CS_Pos in $(Y, \mathcal{G}_p)$ is a $CS_P\delta\mathcal{S}os$ in $(X, \mathcal{F}_P)$.
2. $\delta\mathcal{P}$-continuous (briefly, $CS_P\delta\mathcal{P}Cts$) if the inverse image of every CS_Pos in $(Y, \mathcal{G}_p)$ is a $CS_P\delta\mathcal{P}os$ in $(X, \mathcal{F}_P)$.
3. e-continuous (briefly, CS_PeCts) if the inverse image of every CS_Pos in $(Y, \mathcal{G}_p)$ is a CS_Peos in $(X, \mathcal{F}_P)$.
4. e^*-continuous (briefly, CS_Pe^*Cts) if the inverse image of every CS_Pos in $(Y, \mathcal{G}_p)$ is a CS_Pe^*os in $(X, \mathcal{F}_P)$.
5. a-continuous (briefly, CS_PaCts) if the inverse image of every CS_Pos in $(Y, \mathcal{G}_p)$ is a CS_Paos in $(X, \mathcal{F}_P)$.

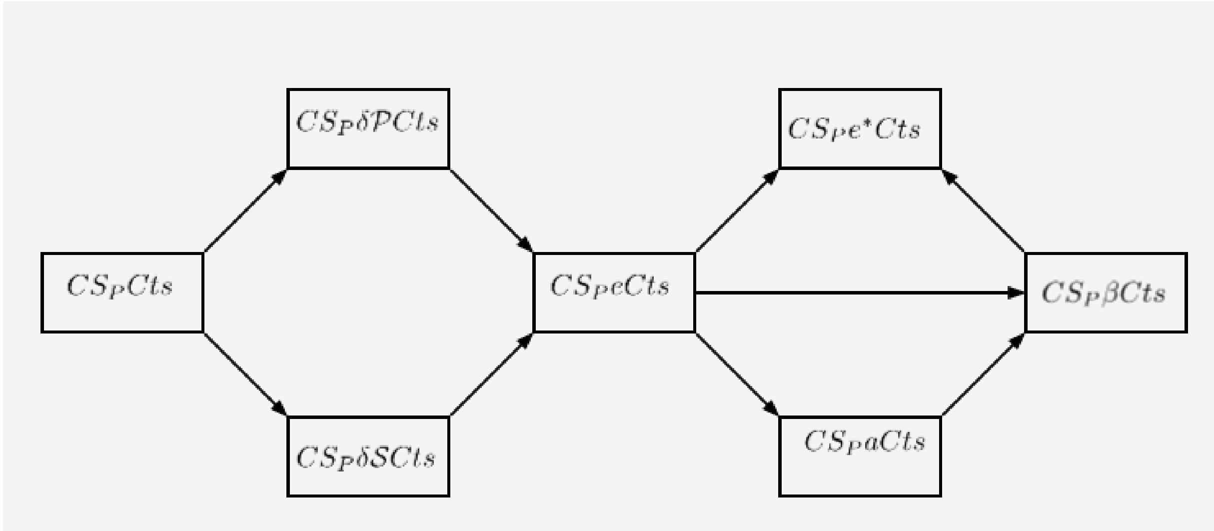
Proposition 3.1 *The statements are hold but the converse does not true. Every*

1. CS_PCts is a $CS_P\delta\mathcal{S}Cts$.
2. CS_PCts is a $CS_P\delta\mathcal{P}Cts$.
3. $CS_P\delta\mathcal{S}Cts$ is a CS_PeCts.
4. $CS_P\delta\mathcal{P}Cts$ is a CS_PeCts.
5. CS_PeCts is a CS_Pe^*Cts.
6. CS_PeCts is a CS_PaCts.
7. CS_PaCts is a $CS_P\beta Cts$.
8. $CS_P\beta Cts$ is a CS_Pe^*Cts.

Proof.

1. Let $\mathfrak{M}$ be a CS_Pos in Y. Since f is CS_PCts, $f^{-1}(\mathfrak{M})$ is CS_Pos in X. Since all CS_Pos are $CS_P\delta\mathcal{S}os$, $f^{-1}(\mathfrak{M})$ is $CS_P\delta\mathcal{S}os$ in X. Hence f is a $CS_P\delta\mathcal{S}Cts$.
2. Let $\mathfrak{M}$ be a CS_Pos in Y. Since f is CS_PCts, $f^{-1}(\mathfrak{M})$ is CS_Pos in X. Since all CS_Pos are $CS_P\delta\mathcal{P}os$, $f^{-1}(\mathfrak{M})$ is $CS_P\delta\mathcal{P}os$ in X. Hence f is a $CS_P\delta\mathcal{P}Cts$.
3. Let $\mathfrak{M}$ be a CS_Pos in Y. Since f is $CS_P\delta\mathcal{S}Cts$, $f^{-1}(\mathfrak{M})$ is a $CS_P\delta\mathcal{S}os$ in X. Since every $CS_P\delta os$ is a CS_Peos, $f^{-1}(\mathfrak{M})$ is a CS_Peos in X. Hence f is a CS_PeCts.
4. Let $\mathfrak{M}$ be a CS_Pos in Y. Since f is $CS_P\delta\mathcal{P}Cts$, $f^{-1}(\mathfrak{M})$ is a $CS_P\delta\mathcal{P}os$ in X. Since every $CS_P\delta\mathcal{P}os$ is a CS_Peos, $f^{-1}(\mathfrak{M})$ is a CS_Peos in X. Hence f is a CS_PeCts.
5. Let $\mathfrak{M}$ be a CS_Pos in Y. Since f is CS_PeCts, $f^{-1}(\mathfrak{M})$ is a CS_Peos in X. Since every CS_Peos is a CS_Pe^*os , $f^{-1}(\mathfrak{M})$ is a CS_Pe^*os in X. Hence f is a CS_Pe^*Cts.
6. Let $\mathfrak{M}$ be a CS_Pos in Y. Since f is CS_PeCts, $f^{-1}(\mathfrak{M})$ is a CS_Paos in X. Since every CS_Peos is a CS_Paos , $f^{-1}(\mathfrak{M})$ is a CS_Paos in X. Hence f is a CS_PaCts.
7. Let $\mathfrak{M}$ be a CS_Pos in Y. Since f is CS_PaCts, $f^{-1}(\mathfrak{M})$ is a $CS_P\beta os$ in X. Since every CS_Paos is a $CS_P\beta os$, $f^{-1}(\mathfrak{M})$ is a $CS_P\beta os$ in X. Hence f is a $CS_P\beta Cts$.
8. Let $\mathfrak{M}$ be a CS_Pos in Y. Since f is $CS_P\beta Cts$, $f^{-1}(\mathfrak{M})$ is a CS_Pe^*os in X. Since every $CS_P\beta os$ is a CS_Pe^*os , $f^{-1}(\mathfrak{M})$ is a CS_Pe^*os in X. Hence f is a CS_Pe^*Cts.

Remark 3.1 *We obtain the following diagram from the results we discussed above and justified from the following examples.*



Example 3.1 *Let X be a non-empty set and let $\mathcal{F}_P = \{\hat{0},\hat{1},\mu_1,\mu_2,\mu_3\},\mathcal{F}'_p = \{\hat{0},\hat{1},\mu_4,\mu_5,\mu_6,\mu_7,\mu_8,\mu_9\}$ be two P-cubic topologies on X where $\mu_1 = \langle[0.2,0.4],0.3\rangle,\mu_2 = \langle[0.5,0.7],0.6\rangle,\mu_3 = \langle[0.8,0.9],0.8\rangle,\mu_4 = \langle[0.4,0.6],0.5\rangle,\mu_5 = \langle[0.7,0.9],0.8\rangle,\mu_6 = \langle[0.1,0.5],0.7\rangle,\mu_7 = \langle[0.1,0.2],0.2\rangle,\mu_8 = \langle[0.3,0.4],0.4\rangle,\mu_9 = \langle[0.6,0.8],0.7\rangle$. Define an idendity mapping $f_p:(X,\mathcal{F}_P) \to (X,\mathcal{F}'_p)$. Here f_p is $CS_P\delta\mathcal{P}Cts$ but not a CS_PCts, since μ_4 is $CS_P\delta\mathcal{S}os$ but not CS_Pos in $(X,\mathcal{F}_P)$.*

Example 3.2 *Let X be a non-empty set and let $\mathcal{F}_P = \{\hat{0},\hat{1},\mu_1,\mu_2,\mu_3\},\mathcal{F}'_p = \{\hat{0},\hat{1},\mu_4,\mu_5,\mu_6,\mu_7,\mu_8,\mu_9\}$ be two P-cubic topologies on X where $\mu_1 = \langle[0.2,0.4],0.3\rangle,\mu_2 = \langle[0.5,0.7],0.6\rangle,\mu_3 = \langle[0.8,0.9],0.8\rangle,\mu_4 = \langle[0.4,0.6],0.5\rangle,\mu_5 = \langle[0.7,0.9],0.8\rangle,\mu_6 = \langle[0.1,0.5],0.7\rangle,\mu_7 = \langle[0.1,0.2],0.2\rangle,\mu_8 = \langle[0.3,0.4],0.4\rangle,\mu_9 = \langle[0.6,0.8],0.7\rangle$. Define an idendity mapping $f_p:(X,\mathcal{F}_P) \to (X,\mathcal{F}'_p)$. Here f_p is $CS_P\delta SCts$ but not a CS_PCts, since μ_9 is $CS_P\delta\mathcal{S}os$ but not CS_Pos in $(X,\mathcal{F}_P)$.*

Example 3.3 *Let X be a non-empty set and let $\mathcal{F}_P = \{\hat{0},\hat{1},\mu_1,\mu_2,\mu_3\},\mathcal{F}'_p = \{\hat{0},\hat{1},\mu_4,\mu_5,\mu_6,\mu_7,\mu_8,\mu_9\}$ be two P-cubic topologies on X where $\mu_1 = \langle[0.2,0.4],0.3\rangle,\mu_2 = \langle[0.5,0.7],0.6\rangle,\mu_3 = \langle[0.8,0.9],0.8\rangle,\mu_4 = \langle[0.4,0.6],0.5\rangle,\mu_5 = \langle[0.7,0.9],0.8\rangle,\mu_6 = \langle[0.1,0.5],0.7\rangle,\mu_7 = \langle[0.1,0.2],0.2\rangle,\mu_8 = \langle[0.3,0.4],0.4\rangle,\mu_9 = \langle[0.6,0.8],0.7\rangle$. Define an idendity mapping $f_p:(X,\mathcal{F}_P) \to (X,\mathcal{F}'_p)$. Here f_p is CS_PeCts but not a $CS_P\delta\mathcal{P}Cts$, since μ_9 is CS_Peos but not $CS_P\delta\mathcal{P}os$ in $(X,\mathcal{F}_P)$.*

Example 3.4 *Let X be a non-empty set and let $\mathcal{F}_P = \{\hat{0},\hat{1},\mu_1,\mu_2,\mu_3\},\mathcal{F}'_p = \{\hat{0},\hat{1},\mu_4,\mu_5,\mu_6,\mu_7,\mu_8,\mu_9\}$ be two P-cubic topologies on X where $\mu_1 = \langle[0.2,0.4],0.3\rangle,\mu_2 = \langle[0.5,0.7],0.6\rangle,\mu_3 = \langle[0.8,0.9],0.8\rangle,\mu_4 = \langle[0.4,0.6],0.5\rangle,\mu_5 = \langle[0.7,0.9],0.8\rangle,\mu_6 = \langle[0.1,0.5],0.7\rangle,\mu_7 = \langle[0.1,0.2],0.2\rangle,\mu_8 = \langle[0.3,0.4],0.4\rangle,\mu_9 = \langle[0.6,0.8],0.7\rangle$. Define an idendity mapping $f_p:(X,\mathcal{F}_P) \to (X,\mathcal{F}'_p)$. Here f_p is CS_PeCts but not a $CS_P\delta SCts$, since μ_5 is CS_Peos but not $CS_P\delta\mathcal{S}os$ in $(X,\mathcal{F}_P)$.*

Example 3.5 *Let X be a non-empty set and let $\mathcal{F}_P = \{\hat{0},\hat{1},\mu_1,\mu_2,\mu_3\},\mathcal{F}'_p = \{\hat{0},\hat{1},\mu_4,\mu_5,\mu_6,\mu_7,\mu_8,\mu_9\}$ be two P-cubic topologies on X where $\mu_1 = \langle[0.2,0.4],0.3\rangle,\mu_2 = \langle[0.5,0.7],0.6\rangle,\mu_3 = \langle[0.8,0.9],0.8\rangle,\mu_4 = \langle[0.4,0.6],0.5\rangle,\mu_5 = \langle[0.7,0.9],0.8\rangle,\mu_6 = \langle[0.1,0.5],0.7\rangle,\mu_7 = \langle[0.1,0.2],0.2\rangle,\mu_8 = \langle[0.3,0.4],0.4\rangle,\mu_9 = \langle[0.6,0.8],0.7\rangle$. Define an idendity mapping $f_p:(X,\mathcal{F}_P) \to (X,\mathcal{F}'_p)$. Here f_p is CS_Pe^*Cts but not a CS_PeCts, since μ_6 is CS_Pe^*os but not CS_Peos in $(X,\mathcal{F}_P)$.*

Example 3.6 *Let X be a non-empty set and let $\mathcal{F}_P = \{\hat{0},\hat{1},\mu_1,\mu_2,\mu_3\},\mathcal{F}'_p = \{\hat{0},\hat{1},\mu_4,\mu_5,\mu_6,\mu_7,\mu_8,\mu_9\}$ be two P-cubic topologies on X where $\mu_1 = \langle[0.2,0.4],0.3\rangle,\mu_2 = \langle[0.5,0.7],0.6\rangle,\mu_3 = \langle[0.8,0.9],0.8\rangle,\mu_4 = \langle[0.4,0.6],0.5\rangle,\mu_5 = \langle[0.7,0.9],0.8\rangle,\mu_6 = \langle[0.1,0.5],0.7\rangle,\mu_7 = \langle[0.1,0.2],0.2\rangle,\mu_8 = \langle[0.3,0.4],0.4\rangle,\mu_9 = \langle[0.6,0.8],0.7\rangle$. Define an idendity mapping $f_p:(X,\mathcal{F}_P) \to (X,\mathcal{F}'_p)$. Here f_p is CS_Pe^*Cts but not a $CS_P\beta Cts$, since μ_7 is CS_Pe^*os but not $CS_P\beta os$ in $(X,\mathcal{F}_P)$.*

Example 3.7 *Let X be a non-empty set and let $\mathcal{F}_P = \{\hat{0},\hat{1},\mu_1,\mu_2,\mu_3\},\mathcal{F}'_p = \{\hat{0},\hat{1},\mu_4,\mu_5,\mu_6,\mu_7,\mu_8,\mu_9\}$ be two P-cubic topologies on X where $\mu_1 = \langle[0.2,0.4],0.3\rangle,\mu_2 = \langle[0.5,0.7],0.6\rangle,\mu_3 = \langle[0.8,0.9],0.8\rangle,\mu_4 = \langle[0.4,0.6],0.5\rangle,\mu_5 = \langle[0.7,0.9],0.8\rangle,\mu_6 = \langle[0.1,0.5],0.7\rangle,\mu_7 = \langle[0.1,0.2],0.2\rangle,\mu_8 = \langle[0.3,0.4],0.4\rangle,\mu_9 = \langle[0.6,0.8],0.7\rangle$. Define an idendity mapping $f_p:(X,\mathcal{F}_P) \to (X,\mathcal{F}'_p)$. Here f_p is $CS_P\beta Cts$ but not a CS_PeCts, since μ_6 is $CS_P\beta os$ but not CS_Peos in $(X,\mathcal{F}_P)$.*

Example 3.8 *Let X be a non-empty set and let $\mathcal{F}_P = \{\hat{0},\hat{1},\mu_1,\mu_2,\mu_3\},\mathcal{F}'_p = \{\hat{0},\hat{1},\mu_4,\mu_5,\mu_6,\mu_7,\mu_8,\mu_9\}$ be two P-cubic topologies on X where $\mu_1 = \langle[0.2,0.4],0.3\rangle,\mu_2 = \langle[0.5,0.7],0.6\rangle,\mu_3 = \langle[0.8,0.9],0.8\rangle,\mu_4 = \langle[0.4,0.6],0.5\rangle,\mu_5 = \langle[0.7,0.9],0.8\rangle,\mu_6 = \langle[0.1,0.5],0.7\rangle,\mu_7 = \langle[0.1,0.2],0.2\rangle,\mu_8 = \langle[0.3,0.4],0.4\rangle,\mu_9 = \langle[0.6,0.8],0.7\rangle$. Define an idendity mapping $f_p:(X,\mathcal{F}_P) \to (X,\mathcal{F}'_p)$. Here f_p is $CS_P\beta Cts$ but not a CS_PaCts, since μ_8 is $CS_P\beta os$ but not CS_Paos in $(X,\mathcal{F}_P)$.*

Example 3.9 *Let X be a non-empty set and let $\mathcal{F}_P = \{\hat{0},\hat{1},\mu_1,\mu_2,\mu_3\},\mathcal{F}'_p = \{\hat{0},\hat{1},\mu_4,\mu_5,\mu_6,\mu_7,\mu_8,\mu_9\}$ be two P-cubic topologies on X where $\mu_1 = \langle[0.2,0.4],0.3\rangle,\mu_2 = \langle[0.5,0.7],0.6\rangle,\mu_3 = \langle[0.8,0.9],0.8\rangle,\mu_4 = \langle[0.4,0.6],0.5\rangle,\mu_5 = \langle[0.7,0.9],0.8\rangle,\mu_6 = \langle[0.1,0.5],0.7\rangle,\mu_7 = \langle[0.1,0.2],0.2\rangle,\mu_8 = \langle[0.3,0.4],0.4\rangle,\mu_9 = \langle[0.6,0.8],0.7\rangle$. Define an idendity mapping $f_p:(X,\mathcal{F}_P) \to (X,\mathcal{F}'_p)$. Here f_p is CS_PeCts but not a CS_PaCts, since μ_8 is CS_Peos but not CS_Paos in $(X,\mathcal{F}_P)$.*

Theorem 3.1 *A map $f:(X,\mathcal{F}_P) \to (Y,\mathcal{G}_p)$ is CS_PeCts iff the inverse image of each CS_Pcs in Y is CS_Pecs in X.*

Proof. Let $\mathfrak{M}$ be a CS_Pcs in Y. This implies $\mathfrak{M}^c$ is CS_Pos in Y. Since f is CS_PeCts, $f^{-1}(\mathfrak{M}^c)$ is CS_Peos in X. Since $f^{-1}(\mathfrak{M}^c) = ((f^{-1}\mathfrak{M}))^c$, $f^{-1}(\mathfrak{M})$ is a CS_Pecs in X.

Conversely, let $\mathfrak{M}$ be a CS_Pcs in Y. Then $\mathfrak{M}^c$ is a CS_Pos in Y. By hypothesis $f^{-1}(\mathfrak{M}^c)$ is CS_Peos in X. Since $f^{-1}(\mathfrak{M}^c) = ((f^{-1}\mathfrak{M}))^c$, $(f^{-1}\mathfrak{M}))^c$ is a CS_Peos in X. Therefore $f^{-1}(\mathfrak{M})$ is a CS_Pecs in X. Hence f is CS_PeCts.

Definition 3.2 *A CS_Pt $(X,\mathcal{F}_P)$ is said to be $CS_PeU_{\frac{1}{2}}$ (in short $CS_PeU_{\frac{1}{2}}$)-space, if every CS_Peos in X is a CS_Pos in X.*

Theorem 3.2 *Let $f:(X,\mathcal{F}_P) \to (Y,\mathcal{G}_p)$ be a CS_PeCts, then f is a CS_PCts if X is a $CS_PeU_{\frac{1}{2}}$-space.*

Proof. Let $\mathfrak{M}$ be a CS_Pos in Y. Then $f^{-1}(\mathfrak{M})$ is a CS_Peos in X, by hypothesis. Since X is a $CS_PeU_{\frac{1}{2}}$-space, $f^{-1}(\mathfrak{M})$ is a CS_Pos in X. Hence f is a CS_PeCts.

Theorem 3.3 *Let $f:(X,\mathcal{F}_P) \to (Y,\mathcal{G}_p)$ be a CS_PeCts map and $g:(Y,\mathcal{G}_p) \to (Z,\mathcal{E}_p)$ be a CS_PCts, then $g \circ f:(X,\mathcal{F}_P) \to (Z,\mathcal{E}_p)$ is a CS_PeCts.*

Proof. Let $\mathfrak{M}$ be a CS_Pos in Z. Then $g^{-1}(\mathfrak{M})$ is a CS_Pos in Y, by hypothesis. Since f is a CS_PeCts map, $f^{-1}(g^{-1}(\mathfrak{M})))$ is a CS_Peos in X. Hence $g \circ f$ is a CS_PeCts map.

Theorem 3.4 *Let $f:(X,\mathcal{F}_P) \to (Y,\mathcal{G}_p)$ be a CS_PeCts map. Then the following conditions are hold.*

i. $f(CS_Pecl(\mathfrak{M})) \leq CS_Pcl(f(\mathfrak{M}))$, for all CS_Pcs $\mathfrak{M}$

in X.

ii. $CS_P ecl(f^{-1}\mathfrak{M}) \leq f^{-1}(CS_P cl\mathfrak{M})$, for all $CS_P cs$ $\mathfrak{M}$ in Y.

Proof. (i) Since $CS_P ecl(f(\mathfrak{M}))$ is a $CS_P ecs$ in Y and f is $CS_P eCts$, then $f^{-1}(CS_P ecl(f(\mathfrak{M})))$ is $CS_P ec$ in Y. Now, since $\mathfrak{M} \leq f^{-1}(CS_P cl(f(\mathfrak{M})))$, $CS_P ecl(\mathfrak{M}) \leq f^{-1}(CS_P ecl(f(\mathfrak{M})))$. Therefore, $f(CS_P ecl(\mathfrak{M})) \leq CS_P cl(f(\mathfrak{M}))$.

(ii) By replacing $\mathfrak{M}$ with $f^{-1}(\mathfrak{M})$ in (i), we obtain $f(CS_P ecl(f^{-1}\mathfrak{M})) \leq CS_P cl(f(f^{-1}\mathfrak{M})) \leq CS_P cl\mathfrak{M}$. Hence, $CS_P ecl(f^{-1}\mathfrak{M}) \leq f^{-1}(CS_P cl\mathfrak{M})$.

Remark 3.2 *If f is $CS_P eCts$, then*

1. $f(CS_P ecl(\mathfrak{M}))$ is not necessarily equal to $CS_P cl(f(\mathfrak{M}))$ where $(\mathfrak{M}) \in X$.
2. $CS_P ecl(f^{-1}\mathfrak{M})$ is not necessarily equal to $f^{-1}(CS_P cl\mathfrak{M})$ where $\mathfrak{M} \in Y$.

Theorem 3.5 *f is $CS_P eCts$ iff $f^{-1}(CS_P int(\mathfrak{M})) \leq CS_P eint(f^{-1}(\mathfrak{M}))$, for all $CS_P cs$ $\mathfrak{M}$ in Y.*

Proof. If f is $CS_P eCts$ and $\mathfrak{M} \in Y$. $CS_P int(\mathfrak{M})$ is $CS_P os$ in Y and hence, $f^{-1}(CS_P int(\mathfrak{M}))$ is $CS_P eos$ in X. Therefore $CS_P eint(f^{-1}(CS_P eint(\mathfrak{M}))) = f^{-1}(CS_P int(\mathfrak{M}))$. Also, $CS_P int(\mathfrak{M}) \leq \mathfrak{M}$, implies that $f^{-1}(CS_P int(\mathfrak{M})) \leq f^{-1}(\mathfrak{M})$. Therefore $CS_P eint(f^{-1}(CS_P int(\mathfrak{M}))) \leq CS_P eint(f^{-1}(\mathfrak{M}))$. That is $f^{-1}(CS_P int(\mathfrak{M})) \leq CS_P eint\ (f^{-1}(\mathfrak{M}))$.

Conversely, let $f^{-1}(CS_P int(\mathfrak{M})) \leq CS_P eint(f^{-1}(\mathfrak{M}))$ for all subset $\mathfrak{M}$ of Y. If $\mathfrak{M}$ is $CS_P os$ in Y, then $CS_P int(\mathfrak{M}) = \mathfrak{M}$. By assumption, $f^{-1}(CS_P int(\mathfrak{M})) \leq CS_P eint(f^{-1}(\mathfrak{M}))$. Thus $f^{-1}(\mathfrak{M}) \leq CS_P eint(f^{-1}(\mathfrak{M}))$. But $CS_P eint(f^{-1}(\mathfrak{M})) \leq f^{-1}(\mathfrak{M})$. Therefore $CS_P eint(f^{-1}(\mathfrak{M})) = f^{-1}(\mathfrak{M})$. That is, $f^{-1}(\mathfrak{M})$ is $CS_P eos$ in X, for all $CS_P os$ $\mathfrak{M}$ in Y. Therefore f is $CS_P eCts$ on X.

Remark 3.3 *If f is $CS_P ects$, then $CSeint(f^{-1}(A))$ is not necessarily equal to $f^{-1}(CS_i nt(A))$ where $A \in Y$.*

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